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A general equilibrium model of statistical discrimination

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Abstract

We study a general equilibrium model with endogenous human capital formation in which *ex ante* identical groups may be treated asymmetrically in equilibrium. The interaction between an informational externality and general equilibrium effects creates incentives for groups to specialize. Discrimination may arise even if the corresponding model with a single group has a unique equilibrium. The dominant group gains from discrimination, rationalizing why a majority may be reluctant to eliminate discrimination. The model is also consistent with "reverse discrimination" as a remedy against discrimination since it may be necessary to decrease the welfare of the dominant group to achieve parity.

JEL classification: D82; J71

Keywords: Statistical discrimination; Inequality; Specialization; Human capital

1. Introduction

This paper studies a competitive model that can rationalize group inequalities as a result of statistical discrimination. Two distinguishable groups have identical distributions of productive characteristics, but may in equilibrium specialize. An equilibrium where groups specialize is characterized by differences in human capital investments, average wages and job assignments.

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Unlike the previous literature on statistical discrimination there is a conflict of interest between groups in our model. Discrimination may be interpreted as one group exploiting the other by designating them as "cheap labor" in an unskilled job, which under quite general circumstances increases the average productivity of workers in the dominant group.

While thinking of discrimination in terms of exploitation seems natural to us, the previous literature on statistical discrimination has followed another path. Models differ a lot in details, but discrimination between identical groups is usually rationalized as a coordination failure. To generate discrimination in this way it suffices to construct a model with multiple equilibria. Discrimination is then explained as one group coordinating on a bad equilibrium and the rest of the economy being in a better equilibrium.

When discrimination is explained as pure coordination, it does not matter whether groups are competing for jobs in the same labor market or are living on separate "islands". That is, groups can be treated separately. This modelling strategy has been so dominant that separability between groups sometimes is taken to be a defining feature of the theory of statistical discrimination.

Models where statistical discrimination is a coordination problem are very tractable, an obvious advantage. However, the tractability comes at a cost of some implausible consequences. The dominant group would have nothing to lose if the disadvantaged group could solve the coordination failure, suggesting that economic policies aimed at excluding groups from certain professions (as in the US during the pre-civil rights era, in South Africa during the apartheid regime, and in many Southeast Asian countries today) would be irrational. Moreover, since parity can be achieved without harm to the dominant group one wonders how reverse discrimination would arise in a world where the problem is coordination.

While our model in many ways is closely related to other models of statistical discrimination, it is *not* a model of different groups coordinating on different equilibria. Discrimination can occur also if the model has a unique symmetric equilibrium. There is still an element of a self-confirming prophesy in that the roles of the groups may be reversed in different equilibria and that there always exists a symmetric equilibrium. The difference is that, in an equilibrium with group inequalities, the disadvantaged group cannot re-coordinate on a better equilibrium without a simultaneous re-coordination (on a worse outcome) by the other group.

The dominant group always gains from discrimination, explaining resistance towards measures intended to eliminate economic discrimination as well as why it may be in the self-interest of a dominant group to institutionalize discrimination.

1.1. *Related literature*

There is a large literature on statistical discrimination following the seminal contributions by Arrow [4] and Phelps [15]. One strand assumes exogenous differences in the precision of information, which creates a rationale for firms to use "irrelevant" group characteristics.[1] The other major strand assumes no exogenous differences. Instead, non-trivial choices by workers, typically pre-market investments in human capital, are introduced which generates a rationale to condition on group identity if workers from different groups behave differently in equilibrium.[2] Our work falls into this second category.

Our model borrows some properties from Arrow [4] and Coate and Loury [5]. Like in Arrow's model (but unlike Coate and Loury's) the labor market is competitive. Arrow, however, does not explicitly derive how incentives to invest depend on wages and here we borrow the human capital investment model and the information technology from Coate and Loury to close the model.

2. The model

2.1. The economic environment

2.1.1. Investments in human capital

There are two firms and a continuum of workers with mass normalized to unity. Each worker belongs to one of two identifiable groups, B or W and we denote by λ^J the respective fraction in the population for $J = B, W$. Prior to entering the labor market each worker makes a binary human capital investment decision. A worker either invests in her human capital and becomes a *qualified* worker, or the worker does not invest. If a worker invests, she incurs cost c which is distributed over $[\underline{c}, \bar{c}] \subseteq R$ according to a continuous and strictly increasing cumulative distribution $G(c)$.

Workers are risk neutral with payoffs that are additively separable in income and the cost of investment and do not care directly about task assignments. That is, a worker with cost c who invests and get a wage w gets utility $w - c$, while a worker who does not invest get utility w.

2.1.2. Production technology

To generate output, firms need workers performing two tasks, a *complex task* and a *simple task*. Only qualified workers are able to perform the complex task while all workers are able to perform the simple task. The effective input of labor in the complex task, C, is thus taken to be the quantity of *qualified* workers employed in the complex task and the input of labor in the simple task, S, is the quantity of workers (of both types) employed in the task. Output is given by $y(C, S)$ where $y : R_+^2 \to R_+$ is a production function that satisfies the following assumptions:

A1. y is concave and strictly increasing in both arguments.

[1] As in Phelps [15], Aigner and Cain [1], Cornell and Welch [6], Lundberg and Startz [10], and Oettinger [14].

[2] Examples include Arrow [4], Spence [16], Akerlof [2], Coate and Loury [5] and Foster and Vohra [8]. Similar in spirit are models deriving unequal outcomes from search frictions (see [3,11]).

A2. y is twice continuously differentiable in both arguments over R^2_{++}.
A3. y satisfies constant returns to scale.

2.1.3. Information technology

Employers cannot observe qualifications, but do observe a signal $\theta \in [0, 1]$, distributed according to density f_q if the worker is qualified and f_u otherwise. Both densities are bounded away from zero and, without further loss of generality, $f_q(\theta)/f_u(\theta)$ is increasing in θ. This monotone likelihood ratio property implies that the posterior probability that a worker from group J with signal θ is qualified given prior π^J,

$$p(\theta, \pi^J) \equiv \frac{\pi^J f_q(\theta)}{\pi^J f_q(\theta) + (1 - \pi^J) f_u(\theta)}, \tag{1}$$

is increasing in θ. A high signal is thus good news about a worker. We denote the cumulative distributions by F_q and F_u and assume that a law of large numbers hold so that these are also the realized frequency distributions of signals for qualified and unqualified workers respectively.[3]

2.2. The game

The timing is described in Fig. 1.[4] In the first stage of the game firms post-wages and task assignment rules and workers simultaneously decide on human capital investments. Each worker decides whether to invest and an investment strategy profile in group J is a map $v^J : [\underline{c}, \bar{c}] \to [0, 1]$, where $v^J(c)$ is the proportion of group J workers with cost c that invests.[5] The fraction of investors in group J is $\pi^J = \int v^J(c)\, dG(c)$, which, since c is unobservable and payoff irrelevant to the firm, contains all relevant information about the investment profile v^J for the firms.

Firms may condition wages and job assignments on θ. A strategy for firm i is to select some *wage schedule* $w_i^J : [0, 1] \to R_+$ and a *task assignment rule* $t_i^J : [0, 1] \to [0, 1]$ for each group J, where $t_i^J(\theta)$ is interpreted as the fraction of workers with signal θ employed in the complex task.[6]

[3] See Judd [9] and Feldman and Gilles [7] for general discussions on how to justify such laws of large numbers with continuum random variables. In our model the anonymity of the workers makes it possible to use a simple trick make the frequency distributions coincide with the perceived probability distributions (see [13] for details).

[4] In earlier drafts we considered a more "natural" timing with wage posting carried out after the investment decisions and task assignments carried out after the clearance of the labor market. This makes the strategy sets more complicated, but equilibrium outcomes remain the same.

[5] The formulation assumes that workers with the same cost picks the same probability of investment. This could in equilibrium fail only for a single cost c, which is irrelevant since G has no atoms.

[6] The model would collapse if workers could be made residual claimants on "their contribution to output". Such model would produce a unique equilibrium which would be color-blind and efficient. Our exact specification is consistent with a world where firms can observe output, but not individual

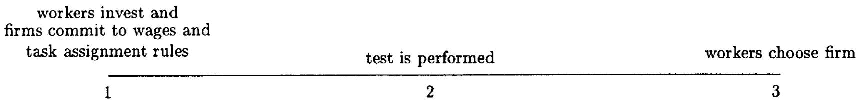

Fig. 1. The timing of the model.

In the second stage nature assigns a signal θ to each worker in accordance with density $f_q(f_u)$ for a worker who invested (did not invest). Workers then observe the posted wages and task assignment rules and decide where to work in the third and final stage.

Investment costs are sunk when workers are comparing wage offers. Hence it is a conditionally strictly dominated strategy not to accept the highest offer. After eliminating strictly dominated firm choice rules the *ex ante* payoff for a worker from group J with investment cost c can thus be written $E_q[\max\{w_1^J(\theta), w_2^J(\theta)\}] - c$ for a worker who invests and $E_u[\max\{w_1^J(\theta), w_2^J(\theta)\}]$ for a worker who does not, where $E_q(E_u)$ is the expectation with respect to $f_q(f_u)$.

3. Color-blind equilibria

As a benchmark we first consider equilibria where both groups invest at the same rate and where firms ignore the payoff irrelevant group characteristic (equivalent to model without the group characteristic). We look for Nash equilibria that satisfy the additional requirement that workers choose firms in a sequentially rational manner after any history of play. Such equilibria are perfect Bayesian, but since beliefs are irrelevant for the optimal firm choice in the end of the game our requirement is really much weaker.

3.1. Equilibrium job assignments and wages

Let $\pi \in [0,1]$ denote the fraction of investors (same for both groups) and imagine that job assignments are carried out by a planner who can choose any task assignment rule $t : [0,1] \to [0,1]$, but takes π as given. The reason for the introduction of a fictitious planner is that constrained efficiency in job assignments is necessary for equilibrium, so this is a convenient way of characterize equilibrium task assignments.

There is total of $\pi f_q(\theta)$ qualified workers and $(1-\pi)f_u(\theta)$ unqualified workers with signal θ. Hence, there are $t(\theta)\pi f_q(\theta)$ "units" of labor in the complex task and (since all workers are equally productive in the simple task) $[1 - t(\theta)][\pi f_q(\theta) + (1-\pi)f_u(\theta)]$ "units" of labor in the simple task with signal θ, so the total inputs of

(*footnote continued*)
productivities. Whereas this is admittedly crude, what is *qualitatively* needed is that the "pre-market signal" θ matters for expected payoffs. We are currently exploring richer contracting environments where this is the case.

labor in the two tasks are

$$C = \int_0^1 t(\theta)\pi f_q(\theta)\,d\theta,$$
$$S = \int_0^1 [1 - t(\theta)][\pi f_q(\theta) + (1-\pi)f_u(\theta)]\,d\theta. \tag{2}$$

Since (1) is increasing in the signal it is without loss of generality to focus on rules $t(\cdot)$ with a threshold property, where workers with signals above the threshold are assigned to the complex task and workers with lower signals are assigned to the simple task. Given a threshold θ' the quantity of *qualified* workers with signals above the threshold is $\pi(1 - F_q(\theta'))$ and the quantity of workers (qualified and unqualified) with signals below the threshold is $\pi F_q(\theta') + (1-\pi)F_u(\theta')$. Output is thus maximized with a threshold solving,

$$\max_{\theta' \in [0,1]} y(\pi[1 - F_q(\theta')], \pi F_q(\theta') + [1-\pi]F_u(\theta')). \tag{3}$$

Let $\theta(\pi)$ be any solution to (3) and, with some abuse of notation, define

$$C(\pi) \equiv \pi[1 - F_q(\theta(\pi))], \qquad S(\pi) \equiv \pi F_q(\theta(\pi)) + (1-\pi)F_u(\theta(\pi)), \tag{4}$$

which are the effective factor inputs in the complex and simple task, respectively, given task assignments in accordance with a threshold rule with cutoff $\theta(\pi)$.

We call a strategy profile a *continuation equilibrium* if all equilibrium conditions except the requirement that investments are best responses to wages are satisfied. Our first result states that wages are given by expected marginal products and job assignments are constrained efficient in any continuation equilibrium.

Proposition 1. *Suppose that a fraction* π *of the workers invest and that* $\theta(\pi)$ *is a solution to* (3). *Then there exists a continuation equilibrium where both firms offer wages*

$$w(\theta;\pi) = \begin{cases} \frac{\partial y(C(\pi),S(\pi))}{\partial S} & \text{for } \theta < \theta(\pi), \\ p(\theta,\pi)\frac{\partial y(C(\pi),S(\pi))}{\partial C} & \text{for } \theta \geqslant \theta(\pi), \end{cases} \tag{5}$$

and where a worker is assigned to the complex task if and only if $\theta \geqslant \theta(\pi)$. *Moreover, in any continuation equilibrium where a fraction* π *of the workers invest the wage schedule posted by* i, $w_i(\theta)$, *must agree with* (5) *for almost all* $\theta \in [0,1]$ *for each firm* i.

Proposition 1 implies:

Corollary 1. *Equilibrium wages are unique up to deviations on sets of signals with measure zero.*

This is obvious if $\theta(\pi)$ is uniquely defined. In case of multiple solutions to (3), constant returns to scale implies that the marginal products are the same evaluated at any solution.

3.2. Equilibrium human capital investments

The final equilibrium condition is that investments are best responses to the wages, implying that a worker invests if and only if the gain in expected earnings is higher than the cost c. We refer to the gain in earnings as *the incentive to invest* and for wages consistent with a continuation equilibrium where a fraction π invests we use direct substitution from (5) to write this as

$$\begin{aligned} I(\pi) &= \int_0^1 w(\theta;\pi) f_q(\theta)\, d\theta - \int_0^1 w(\theta;\pi) f_u(\theta)\, d\theta \\ &= \frac{\partial y(C(\pi), S(\pi))}{\partial S}[F_q(\theta(\pi)) - F_u(\theta(\pi))] \\ &\quad + \frac{\partial y(C(\pi), S(\pi))}{\partial C}\int_{\theta(\pi)}^1 p(\theta,\pi)[f_q(\theta) - f_u(\theta)]\, d\theta. \end{aligned} \tag{6}$$

The fraction of workers that gain from investing is thus $G(I(\pi))$. In a Nash equilibrium, firms have rational expectations about the fraction of investors and investment behavior must be rational given wages, so the equilibria are fully characterized as the solutions to

$$\pi = G(I(\pi)). \tag{7}$$

Hence, the fraction of investors in any equilibrium is a solution to (7) and from any solution to (7) we can construct wage schedules, task assignments and investment rules consistent with equilibrium.[7]

4. Asymmetric equilibria

We now allow groups to invest at different rates and now let $\boldsymbol{\pi} = (\pi^B, \pi^W)$ denote the group-specific fractions of investors. The analogue of problem (3) is

$$\begin{aligned} &\max_{\theta^B,\theta^W \in [0,1]^2} y\Bigg(\sum_{J=B,W} \lambda^J \pi^J [1 - F_q(\theta^J)], \\ &\sum_{J=B,W} \lambda^J [\pi^J F_q(\theta^J) + (1-\pi^J) F_u(\theta^J)]\Bigg), \end{aligned} \tag{8}$$

[7] That is, if π solves (7) there is an equilibrium $\{i, \langle w_i, t_i \rangle_{i=1,2}\}$ where $i(c) = e_q$ for all $c < G^{-1}(\pi)$ and $i(c) = e_u$ for all $c > G^{-1}(\pi)$, $w_i(\theta) = w(\theta)$ all θ and $t_i(\theta)$ is a rule with threshold $\theta(\pi)$.

and given a solution $(\theta^B(\pi), \theta^W(\pi))$ to this program we now use similar abuse of notation as in (4) and define

$$\begin{aligned}
C^J(\boldsymbol{\pi}) &\equiv \pi^J[1 - F_q(\theta^J(\boldsymbol{\pi}))],\\
S^J(\boldsymbol{\pi}) &\equiv \pi^J F_q(\theta^J(\boldsymbol{\pi})) + (1-\pi^J)F_u(\theta^J(\boldsymbol{\pi})) \quad \text{for } J = B, W,\\
C(\boldsymbol{\pi}) &= \lambda^B C^B(\boldsymbol{\pi}) + \lambda^W C^W(\boldsymbol{\pi}),\\
S(\boldsymbol{\pi}) &= \lambda^B S^B(\boldsymbol{\pi}) + \lambda^W S^W(\boldsymbol{\pi}).
\end{aligned} \tag{9}$$

The characterization of equilibrium wages and task assignments is the obvious generalization of Proposition 1 and since the proof proceeds step by step as that in the model with a single group, we have omitted the proof.

Proposition 2. *Suppose that a fractions* $\boldsymbol{\pi} = (\pi^B, \pi^W)$ *of the workers invest and that* $(\theta^B(\boldsymbol{\pi}), \theta^W(\boldsymbol{\pi}))$ *solves* (8). *Then there exists a continuation equilibrium where both firms offer wages*

$$w^J(\theta; \boldsymbol{\pi}) = \begin{cases} \frac{\partial y(C(\boldsymbol{\pi}), S(\boldsymbol{\pi}))}{\partial S} & \text{for } \theta < \theta^J(\boldsymbol{\pi}), \\ p(\theta, \pi^J)\frac{\partial y(C(\boldsymbol{\pi}), S(\boldsymbol{\pi}))}{\partial C} & \text{for } \theta \geqslant \theta^J(\boldsymbol{\pi}), \end{cases} \quad \text{for } J = B, W, \tag{10}$$

and assign a worker with characteristics (J, θ) *to the complex task if and only if* $\theta \geqslant \theta^J(\boldsymbol{\pi})$. *Moreover, in any continuation equilibrium where fractions* $\boldsymbol{\pi} = (\pi^B, \pi^W)$ *of the workers invest the wage schedule posted by i for group* J, $w_i^J(\theta)$, *agrees with* (10) *for almost all* $\theta \in [0, 1]$ *for each firm i.*

The incentive to invest given any investment behavior $\boldsymbol{\pi}$ is

$$I^J(\boldsymbol{\pi}) = \frac{\partial y(C(\boldsymbol{\pi}), S(\boldsymbol{\pi}))}{\partial S}[F_q(\theta^J(\boldsymbol{\pi})) - F_u(\theta^J(\boldsymbol{\pi}))] \tag{11}$$

$$+ \frac{\partial y(C(\boldsymbol{\pi}), S(\boldsymbol{\pi}))}{\partial C}\int_{\theta^J(\boldsymbol{\pi})}^1 p(\theta, \pi^J)[f_q(\theta) - f_u(\theta)]\, d\theta, \tag{12}$$

and the fraction of agents in group j who invest is still given by the fraction with investment cost lower than the benefits, so the system of equations that characterize the equilibria is

$$\pi^J = G(I^J(\boldsymbol{\pi})) \quad \text{for } J = B, W. \tag{13}$$

We say that an equilibrium is *discriminatory* whenever $\pi^B \neq \pi^W$, which is consistent with the standard definition of economic discrimination in terms of average wage differentials. Inspection of (10) shows that if $\pi^B < \pi^W$, then the wage is higher for W-workers than for B-workers for each signal (strictly higher for $\theta > \theta^J(\boldsymbol{\pi})$). Moreover, more workers in group W have high signals, so in our stylized model the group with the lower fraction of investors is also the group with the lower average wage.

4.1. Sufficient conditions for unique optimal task assignments

It is notationally convenient to rule out multiple solutions to (8). There is no substantial cost of doing this since multiplicity of solutions to (8) generates nothing qualitatively different (due to Corollary 1). Sufficient conditions are:

Lemma 1. *Suppose that either* (1) *y is quasi-concave and strictly increasing in both arguments and $f_q(\theta)/f_u(\theta)$ is strictly increasing in θ or* (2) *y is strictly quasi-concave and strictly increasing in both arguments. Then, there is a unique $(\theta^B(\boldsymbol{\pi}), \theta^W(\boldsymbol{\pi})) \in [0,1]^2$ that solves* (8) *for any $\boldsymbol{\pi} \gg 0$.*

To understand this it is useful to restate problem (8) as

$$\begin{aligned} &\max_{C^J, S^J} \quad y(\lambda^B C^B + \lambda^W C^W, \lambda^B S^B + \lambda^W S^W) \\ &\text{s.t.} \quad S^J \leqslant \pi^J - C^J + (1-\pi^J) F_u\left(F_q^{-1}\left(\frac{\pi^J - C^J}{\pi^J}\right)\right) \quad \text{for } J = B, W. \end{aligned} \tag{14}$$

The monotone likelihood ratio assumption implies that the right-hand side of the constraint is a concave function of C (strictly concave with the strict monotone likelihood assumption), so the two cases can be illustrated as in Fig. 2.

4.2. The linear model

It is useful to first consider the special case with a linear production function, $y(C,S) = qC + uS$, for some $q > u > 0$. To avoid dealing with correspondences we assume that $f_q(\theta)/f_u(\theta)$ is strictly increasing in θ, so that Lemma 1 applies. The task assignment problem (8) then simplifies to

$$\sum_{J=B,W} \lambda^J \max_{\theta^J \in [0,1]} [q\pi^J(1 - F_q(\theta^J)) + u(\pi^J F_q(\theta^J) + (1-\pi^J) F_u(\theta^J))]. \tag{15}$$

That is, the task assignment problem can be solved separately for each group. To stress the separability we write $\hat{\theta}^J(\pi^J)$ for the unique optimal threshold in group

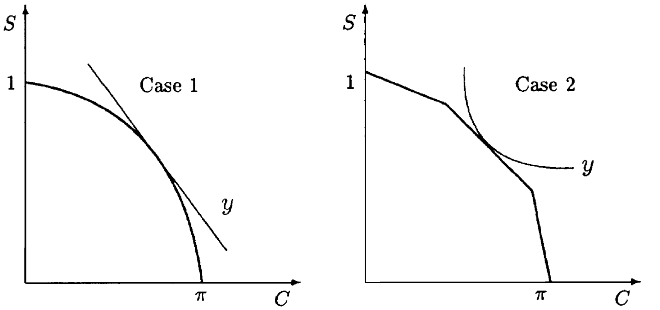

Fig. 2. Sufficient conditions for unique optimal task assignments.

J, which is given by

$$\hat{\theta}^J(\pi^J) = \begin{cases} 1 & \text{if } qp(1,\pi^J) \leqslant u, \\ 0 & \text{if } qp(0,\pi^J) \geqslant u, \\ \text{the unique solution to} & \\ \quad qp(\theta,\pi^J) = u & \text{if } qp(0,\pi^J) < u < qp(1,\pi^J). \end{cases} \tag{16}$$

Equilibrium wages are thus $w^J(\theta;\pi^J) = u$ for $\theta \leqslant \hat{\theta}^J(\pi^J)$ and $w^J(\theta;\pi^J) = qp(\theta,\pi^J)$ for $\theta > \hat{\theta}^J(\pi^J)$ and equilibria are fully described as a pair (π^B, π^W) such that

$$\pi^J = G(\hat{I}^J(\pi^J)) \quad \text{for } J = B, W, \tag{17}$$

where

$$\hat{I}^J(\pi^J) = u[F_q(\hat{\theta}^J(\pi^J)) - F_u(\hat{\theta}^J(\pi^J))] + q\int_{\hat{\theta}^J(\pi^J)}^{1} p(\theta,\pi^J)(f_q(\theta) - f_u(\theta))\, d\theta. \tag{18}$$

Observe that $\hat{I}^J(\pi^J)$ is a composition of continuous functions, which means that existence of equilibria is immediate.

There may be a unique solution to (17), in which case groups must be treated identically in any equilibrium. The more interesting possibility is that (17) may have multiple solutions, in which case there are equilibria with discrimination. This is illustrated in Fig. 3. One shows from (16) that there exists $\underline{\pi} > 0$ such that all workers are assigned to the simple task if $\pi^J \leqslant \underline{\pi}$. Symmetrically, there is some $\bar{\pi} < 1$ such that all workers are assigned to the complex task if $\pi^J \geqslant \bar{\pi}$. If $\pi^J \leqslant \underline{\pi}$ it follows that $\hat{I}^J(\pi^J) = 0$, so if $G(0) \leqslant \underline{\pi}$ it follows that there is a trivial equilibrium where no workers invest (the picture is drawn for $G(0) = 0$). Observe that the incentive to invest is still strictly positive in the (non-empty) range $(\underline{\pi}, 1)$ since the posterior is

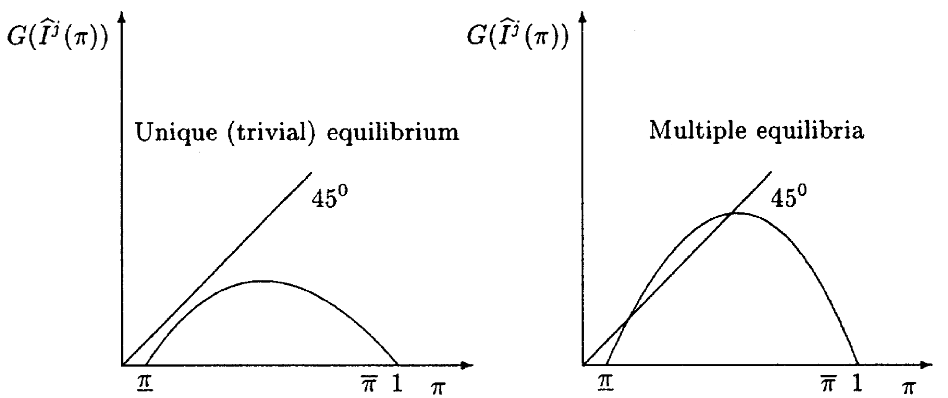

Fig. 3. Illustration of the equilibrium fixed point problem in the linear case.

increasing in the signal, thus generating wage inequality even with all agents allocated to the complex task.

The curve $G(\hat{I}^j(\pi))$ can be moved up and down by changes in G alone. That is, if G first order stochastically dominates G', the whole curve shifts up if G is replaced by G'. Hence, if costs are "sufficiently low" the curve intersects the diagonal line, implying that there then are at least two interior equilibria in addition to the trivial equilibrium.[8]

If there are multiple solutions to (17) each group can have a fraction of investors corresponding to any of these solutions. These equilibria are Pareto rankable.

Proposition 3. *Let π^* be the largest solution to* (17). *Then* $(\pi^B, \pi^W) = (\pi^*, \pi^*)$ *Pareto dominates all other equilibria of the model.*

Discrimination can thus be sustained in the linear model, but only as a pure coordination failure. The separability between groups implies that the "dominant group" would not be affected at all if the discriminated group could somehow re-coordinate on a better equilibrium. This property is shared by the model in Coate and Loury [5] and almost all equilibrium models that can rationalize unequal treatment of identical groups.[9]

Our view is that this separability is a weakness of the theory. Given the long-standing record of economic policies designed to exclude certain groups from high-income professions, it seems that there simply must be some gains from such measures for those that the policies are intended to "protect". Put differently, a rather natural belief is that economic discrimination against blacks has something do with Jim Crow laws in the past, and it seems strange then to perform the analysis within a model where such laws would be irrational. Similarly, the idea of "reverse discrimination" as a remedy for past discrimination appears equally irrational, again suggesting that a richer model is needed.

5. Complementarities

We assume that, in addition to assumptions A1–A3, the production function y satisfies,

A4. y is strictly quasi-concave.

[8] There can be more than two interior equilibria. The easiest way to see that is to consider the (non-generic) case with a continuum of equilibria. Given any f_q, f_u, α, and β we can construct a distribution G that supports a continuum of equilibria as follows. Take an interval $(a,b) \subset [\underline{c}, \bar{c}]$ where B is strictly increasing (such range must exist since B is continuous and strictly positive at any $\underline{\pi} < \pi < 1$). Define $\tilde{B}: (a,b) \to R$ as $\tilde{B}(\pi) = B(\pi)$ for any $\pi \in (a,b)$ and let G be a function satisfying $G^{-1}(\pi) = \tilde{B}(\pi)$ for any $\pi \in (a,b)$, which immediately implies that any $\pi \in (a,b)$ is an equilibrium. It should be intuitive from this that we may construct (more robust) examples where there are k equilibria for any integer k.

[9] The only exception we are aware of is that there are matching and search models where groups cannot be analyzed in separation, see [11].

A5. $\lim_{C \to 0} \frac{\partial y(C,S)}{\partial C} = \infty$ for any $S > 0$ and $\lim_{S \to 0} \frac{\partial y(C,S)}{\partial S} = \infty$ for any $C > 0$.
A6. $y(0,S) = y(C,0) = 0$ for any $C, S > 0$.

Strict quasi-concavity is the qualitatively important assumption, while A5 and A6 are for expositional simplicity. Existence of equilibria can be checked rather easily from the reduced form characterization in (11) and (13).

Proposition 4. *Suppose y satisfies assumptions* A1–A6. *Then there is always at least one symmetric equilibrium.*

5.1. Cross-group effects on incentives

Strict quasi-concavity implies (by Lemma 1) that there is a unique $(\theta^B(\boldsymbol{\pi}), \theta^W(\boldsymbol{\pi}))$ solving (8) whenever $(\pi^B, \pi^W) \gg 0$. Moreover, since output is zero whenever all workers are assigned to the same task the factor ratio,

$$r(\boldsymbol{\pi}) = r(\pi^B, \pi^W) = \frac{\sum_{J=B,W} \lambda^J \pi^J (1 - F_q(\theta^J(\boldsymbol{\pi}))}{\sum_{J=B,W} \lambda^J [\pi^J F_q(\theta^J(\boldsymbol{\pi})) + (1-\pi^J) F_u(\theta^J(\boldsymbol{\pi}))]}, \tag{19}$$

is always well defined. In case of a fully interior solution to (8) the necessary and sufficient conditions for optimality may be expressed as

$$p(\theta^J(\boldsymbol{\pi}), \pi^J) \frac{\partial y(r(\boldsymbol{\pi}), 1)}{\partial C} = \frac{\partial y(r(\boldsymbol{\pi}), 1)}{\partial S}. \tag{20}$$

The crucial observation to be made from (20) is that *the ratio of complex to simple labor is monotonically increasing in the fraction of investors in any group*. To see this, suppose to the contrary that the factor ratio goes down when π^J increases. To satisfy (20) it is then necessary for $\theta^J(\boldsymbol{\pi})$ to decrease in each group. This in turn would, as can be seen in (19), imply that the factor ratio increased, which is a contradiction.[10]

The monotonicity of the factor ratio in investments generates negative cross group effect on incentives.

Proposition 5. *Fix $\pi^J > 0$. Then $I^J(\pi^J, \pi^K)$ is decreasing in π^K over the whole unit interval and strictly decreasing for all $\boldsymbol{\pi}$ such that $\theta^J(\boldsymbol{\pi}) < 1$ and $\theta^K(\boldsymbol{\pi}) > 0$.*

Since these effects are central to the understanding of our model, we now provide a heuristic explanation. Rewrite the equilibrium wage schemes as

$$w^J(\theta; \boldsymbol{\pi}) = \begin{cases} \frac{\partial y(r(\boldsymbol{\pi}),1)}{\partial S} & \text{for } \theta < \theta^J(\boldsymbol{\pi}), \\ p(\theta, \pi^J) \frac{\partial y(r(\boldsymbol{\pi}),1)}{\partial C} & \text{for } \theta \geqslant \theta^J(\boldsymbol{\pi}), \end{cases} \quad \text{for } J = B, W. \tag{21}$$

[10] The proof of Proposition 5 also takes care of the possibility of corner solutions to (8).

Observe that $r(\boldsymbol{\pi})$ increases when π^W increases. Hence, $\partial y(r(\boldsymbol{\pi}), 1)/\partial S$ increases and $p(\theta, \pi^B) \cdot \partial y(r(\boldsymbol{\pi}), 1)/\partial C$ decreases with an increase in investments in group W. As shown in the left panel of Fig. 4, the effect on the wage scheme for B workers is thus that the wage for workers that are employed in the simple (complex) task will increase (decrease), and that the threshold increases. Since F_q (the distribution of signals for a qualified worker) first order stochastically dominates F_u (the distribution of signals for an unqualified worker) the conclusion is that the change unambiguously reduces the incentives to invest for B workers, as asserted in Proposition 5.

To the right in Fig. 4 we have decomposed the effect on incentives for W workers from the same change. The point with the graph is that while the effects from the change in $r(\boldsymbol{\pi})$ remains the same, there is also an "informational effect" which may tend to improve incentives. In the right graph, the shift labeled $\downarrow_a$ identifies the "price effect", which has the same direction it has on B workers. The reason that incentives for W workers *may* still improve is that $p(\theta, \pi^W)$ increases for every θ, which in the graph leads to the shift labeled $\uparrow_b$. The graph is drawn assuming that the increase in $p(\theta, \pi^w)$ is sufficiently large so that the net effect for W workers in the complex task is that the wage increases, but whether this is the case or not depends on π^W and π^B.

However wages of skilled workers' from group W change also because of the "informational effect" $p(\theta, \pi^w)$ which may increase or decrease as π^W increase, depending on the size of π^W. We have drawn the figure assuming that $p(\theta, \pi^w)$ increases and more than compensates the price effect (see $\downarrow_b$), but it need not be the case. The effect on W workers' incentives are therefore not obvious.

This "as if externality" is driven by ordinary price effects, but the informational externality is still crucial. With observable investments, there would be a wage w_q for qualified workers and a wage w_u for unqualified workers. For the same reasons as in our model, an increase (in any group) of the fraction of investors would tend to decrease w_q and increase w_u in equilibrium. However, benefits to invest would be

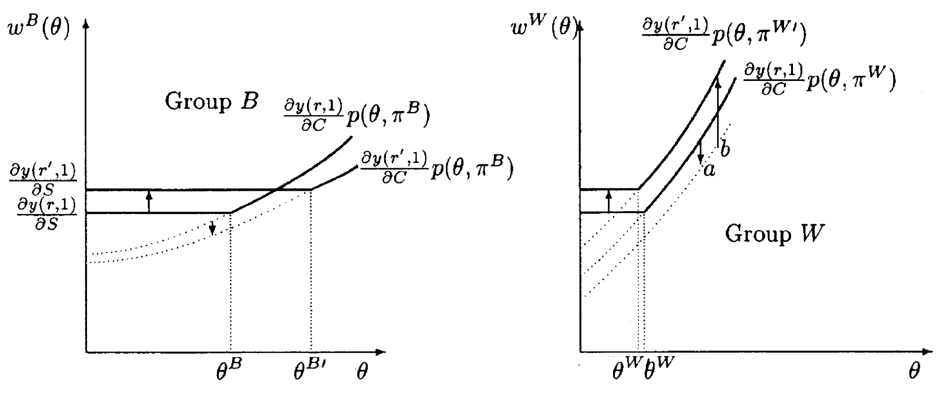

Fig. 4. The effect of an increase in the fraction of investors in group W.

$w_q - w_u$ for both groups. A change in the proportion of qualified workers (in any group) thus affects incentives symmetrically. The informational asymmetry is therefore essential in our model since otherwise no differential treatment can occur in equilibrium.

5.2. When will asymmetric equilibria exist?

To construct an example where group B is discriminated against it suffices to find some $\boldsymbol{\pi} = (\pi^B, \pi^W)$ where $\pi^B < \pi^W$ and $I^B(\boldsymbol{\pi}) < I^W(\boldsymbol{\pi})$. Such an investment profile $\boldsymbol{\pi}$ always exists and, once this is found, any distribution function G such that $\pi^B = G(I^B(\boldsymbol{\pi}))$ and $\pi^W = G(I^W(\boldsymbol{\pi}))$ completes the example.

Parameters interact with each other in a rather complicated way, even in tightly parameterized versions of the model. It is therefore hard to come up with useful sufficient conditions for existence of asymmetric equilibria. The one general result that does provide some information is highly intuitive. To state the result, let G be some distribution function with $G(0) = 0$ and assume the single-group model with distribution G has a non-trivial equilibrium. Define the parametric sequence of distributions G_ψ, where $G_\psi(c) = \psi + (1 - \psi)G(c)$ for every c in the support of G.

Proposition 6. *Fix y, f_q, f_u, λ^B and λ^W. Then there exists $\bar{\psi} > 0$ such that a discriminatory equilibrium exists in the model with cost distribution G_ψ for any $\psi \leqslant \bar{\psi}$.*

The intuition is straightforward: as long as there are not too many agents who get positive utility from the human capital investment it is possible to construct equilibria with all agents from one group in the simple task. This is not possible in any symmetric equilibrium under the assumptions on page 12.

Of the other parameters in the model we suspect that discrimination is easier to sustain the larger is the size difference between groups. We have however not been able to prove this even for the parametric version of the model in Section 6. From the parametric example we know that the precision of the signal affects the likelihood of an asymmetric equilibrium non-monotonically. Very precise signals makes discrimination hard to obtain because prior information gets discounted a lot. If the signal is too uninformative, discrimination is hard to sustain because it gets hard to sustain incentives for any group. The "importance of the complex task" also matters. This is hard to formalize in general, but in Section 6 this is summarized in a single parameter and, not surprisingly, the effects are again non-monotonic (see Section 6.3).

5.3. Gains for the dominant group

The monotone spillover effects also create an incentive to discriminate. If a certain group could choose between a symmetric equilibrium and an equilibrium where the other group is discriminated against they would always choose to discriminate the other group. The result holds also if there are multiple symmetric equilibria given that the discriminatory equilibrium is wisely chosen, but is easiest to state under the assumption that the symmetric equilibrium is unique (in the next section we consider a parameterization where this is always the case):

Proposition 7. *Suppose that there is a unique symmetric equilibrium. Then, in any equilibrium with discrimination, the ex ante utility (before knowing the cost realization) in the group with the higher fraction of investors is higher than that in the symmetric equilibrium.*

While this may seem obvious the reader should note that this is never the case in a model where discrimination is a coordination failure, so the cross-group effects is what makes the model generate this rather natural prediction.

5.4. Gains and losses from specialization

To analyze the impact from groups specializing on society as a whole it is convenient to define the maximized value of output given any particular investment behavior as

$$Y(\boldsymbol{\pi}) \equiv \max_{\theta^B,\theta^W} y\left(\sum_{J=B,W} \lambda^J \pi^J [1 - F_q(\theta^J)], \sum_{J=B,W} \lambda^J F_{\pi^J}(\theta^J)\right). \tag{22}$$

Social surplus in the economy is

$$Y(\boldsymbol{\pi}) - \sum_{J=B,W} \lambda^J \int_{\underline{c}}^{G^{-1}(\pi^J)} cg(c)\, dc. \tag{23}$$

Let $h(\pi^W)$ be a function such that $\lambda^B h(\pi^W) + \lambda^W \pi^W = K$ holds for all π^W in some range for some constant $K > 0$. That is, $(h(\pi^W), \pi^W)$ defines a (linear) locus of group-specific fractions of investors such that the total investments in the economy is held constant.

Proposition 8. *Suppose that* $\pi^B < \pi^W$. *Then*:

1. $\frac{d}{d\pi^W} Y(h(\pi^W), \pi^W) > 0$,
2. $\frac{d}{d\pi^W} (\lambda^B \int_{\underline{c}}^{G^{-1}(h(\pi^W))} cg(c)\, dc + \lambda^W \int_{\underline{c}}^{G^{-1}(\pi^W)} cg(c)\, dc) > 0.$

This result says that output increases with increased specialization and that aggregate investment costs increase with increased specialization. However, nothing guarantees that two *equilibria* have the same total quantity of investors. Proposition 8 is therefore only suggestive about welfare comparisons across equilibria.

Keeping this caveat in mind, the result that output is increasing in the degree of specialization has an intuitive explanation. Some workers are always assigned to the wrong job and this "mismatch" is reduced when groups specialize.[11] That aggregate investment costs increase should be obvious, since investments are transferred from lower to higher cost units.

6. A parametric example

We now parameterize the model by letting $y(C, S) = C^{\alpha}S^{1-\alpha}$ for some $\alpha \in (0, 1)$ and assume that θ is drawn from $\{\theta_L, \theta_H\}$ in accordance with symmetric conditional probability distributions, where $\phi > \frac{1}{2}$ is the probability of drawing θ_H for a qualified worker, and $(1 - \phi)$ is the probability of θ_H for an unqualified worker. We also let the cost c be uniformly distributed over $[\underline{c}, \bar{c}]$. To conserve on space we have omitted all derivations that are used for the numerical solutions, but all details are available in Moro and Norman [13].

6.1. Incentives to specialize

It turns out that symmetric equilibria are unique in the parameterized model provided that $\underline{c} < 0$ (see [13]). Sometimes the symmetric equilibrium is the only equilibrium of the model, but when asymmetric equilibria exist there is typically more than one, even when fixing the roles of the groups. To avoid setwise comparisons we will therefore restrict attention to asymmetric equilibria where all B workers are assigned in the simple task, which is the most extreme form of segregation that can occur in the model.

Fig. 5 shows a rather typical example of such an extreme equilibrium.[12] The dotted line in each figure shows the fixed point equation for the symmetric equilibrium, which would be the relevant one if the groups populated separate economies. The solid lines are the projections of the fixed point equation that are obtained if each group is assuming that the other group will behave in accordance with the asymmetric equilibrium (π^B, π^W). For each group, the only intersection between the projected fixed point map and the 45° line is at the equilibrium value. Hence, if it is known that W coordinates on an equilibrium where B workers are discriminated against, the only rational response by the B workers is to behave in accordance with that equilibrium.

The main point of this example is to highlight how different our model is from the usual coordination-based model with separable groups. It is the interaction between that is actually the driving force for the inequalities rather than pure coordination. The reason that the B workers invest so little in human capital is that the existence of the highly qualified group W destroys the incentives. Symmetrically, the existence of

[11] This is true also in the linear model, but then there is no particular reason for the *aggregate* quantity of investors to be at any particular level. With curvature in the technology, it is undesirable to vary factor inputs too much, which means that the trade-off summarized in Proposition 8 becomes relevant for efficiency.

[12] The parameterization shown in the figure is $\phi = \frac{2}{3}$, $\alpha = \frac{1}{2}$, $\lambda^W = \frac{1}{2}$ $c \sim U[-0.02, 0.18]$.

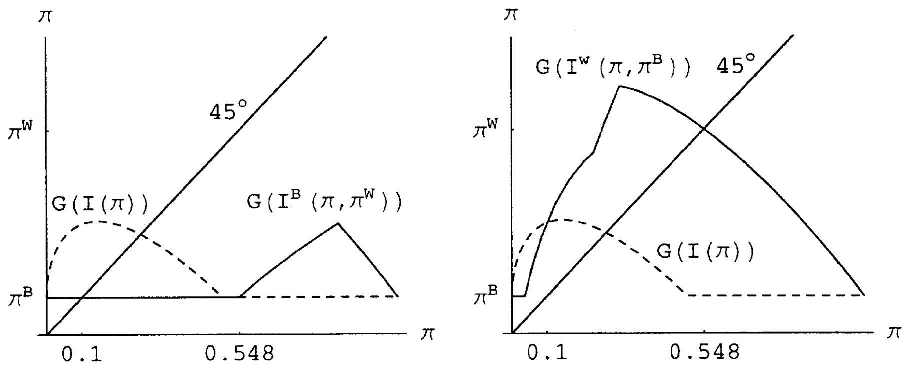

Fig. 5. Equilibrium best responses.

Table 1
Symmetric and asymmetric equilibria, average welfare computed as expected wage less average investment costs

$\phi=\frac{2}{3},\ \alpha=\frac{1}{2},\ \lambda^W=\frac{1}{2}\ c\sim U[-0.02,0.18]$	Discriminatory equilibrium		Symmetric equilibrium
	Group B	Group W	
Equilibrium investment	$\pi^B=0.1$	$\pi^W=0.548$	$\pi=0.269$
Gross incentives to invest	$I^B(\boldsymbol{\pi})=0$	$I^W(\boldsymbol{\pi})=0.090$	$I^J(\pi)=0.034$
Wages	$w^B(\theta_H,\boldsymbol{\pi})=0.307$	$w^W(\theta_H,\boldsymbol{\pi})=0.576$	$w(\theta_H,\pi)=0.380$
	$w^B(\theta_L,\boldsymbol{\pi})=0.307$	$w^W(\theta_L,\boldsymbol{\pi})=0.307$	$w(\theta_L,\pi)=0.279$
Average expected welfare	0.309	0.416	0.320
Expected welfare if invest	$0.307-c$	$0.487-c$	$0.346-c$
Exp. welfare if not invest	0.307	0.397	0.313

the mainly unqualified group B is what creates incentives for high investments in group W.

6.2. Winners and losers

The group with the higher fraction of investors is always better off (ex ante) in an asymmetric equilibrium than in the symmetric equilibrium (Proposition 7), but we have so far not discussed welfare effects for the discriminated group. Workers from the discriminated group are moved away from the more lucrative task, but the wage in the simple task is also changing due to changes in the factor ratio, so the effect is not a priori obvious.

Indeed, the welfare effects for the discriminated group may go either way when comparing the discriminatory equilibrium with the symmetric equilibrium. For the parameterization considered in Fig. 5 it turns out that there is a conflict of interest between the groups. Table 1 summarizes the comparison. The last 4 rows in the table show that the dominant group gains and the discriminated group loses relative to the symmetric equilibrium. In this particular example this is true also conditional on the investment decision. Every worker in group W (B) is thus better (worse) off in the discriminatory equilibrium than in the symmetric equilibrium.[13]

It is also possible that both groups gain from discrimination. The simplest example is if investment costs are so high that nobody invests in the symmetric equilibrium. One such case is when $c \sim U[0.03, 0.23]$ and all other parameters are as in the example in Table 1. In this case the unique symmetric equilibrium is for no worker to invest (which implies zero production). However there is also an asymmetric equilibrium where $(\pi^B, \pi^W) = (0, 0.39)$. Positive production implies that wages are strictly positive for all workers, so the asymmetric equilibrium is beneficial to both groups.

Less trivial examples, where discrimination is beneficial for all workers despite positive production in the symmetric equilibrium, can also be constructed. The general idea is that specialization increases output unless the total number of qualified workers falls (too much) and that the an increased wage in the simple task may make up for the reduced opportunities in the complex task for the discriminated group.

6.3. *A rationale for institutionalized discrimination*

Since the dominant group gains from discrimination our model immediately suggests one rationale for apartheid or other discriminatory measures. We may interpret such a policy as a *coordination device* assuring that the most preferred equilibrium for the group with political control is realized.

There is a second, maybe more interesting, rationale for discriminatory policies in the model. While discrimination is always preferable for the dominant group it may simply not be sustainable in equilibrium. Hence, a law that forbids firms to assign workers from group B to the complex task may make workers from group W better off than in any "laissez faire" equilibrium.

In our model, the smaller is the group that have access to the better job, the less likely it is that is it compatible with equilibrium not to hire workers from the discriminated group to that job. To see this, suppose all B workers are in the simple task and assume that there are *some* workers in group B that are qualified ($\underline{c}<0$). Discrimination then gets harder to sustain, the smaller the group W gets, workers in the complex task become more and more valuable. For small enough λ^W the marginal product in the complex task is sufficiently high for employers to hire B workers to the complex task even if the likelihood that they are qualified is small. This in turn creates better incentives and the discriminatory equilibrium unravels. Hence, our model suggests that segregated labor markets must be supported by apartheid-like legislation if the group in power is small. While this appeals to common sense, we are unaware of any other model of discrimination that generates similar results.

[13] The effective ratio of complex to simple labor may either increase or decrease. If the ratio decreases, the wage in the simple task declines, so W-workers with high c (who do not invest) can in general be made worse off.

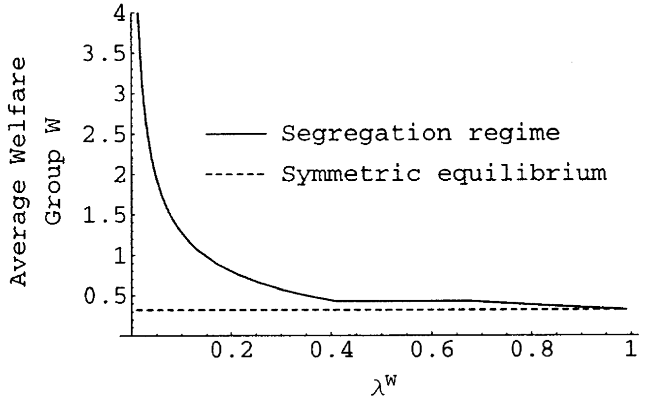

Fig. 6. Average welfare in the dominant group as a function of group size.

While a decrease in λ^W makes a discriminatory equilibrium less likely, the gains from having exclusive rights to the complex task for group W increase as λ^W decreases. In Fig. 6 we illustrate this in an example where we vary the relative group size λ^W and keep all other parameters as in Table 1. The figure plots the average payoff for a worker in group W as a function of λ^W under the assumption that workers from group B are all assigned to the simple task. Since this is incompatible with equilibrium when λ^W is small we refer to this as the payoff in the "segregation regime".[14] The basic point with the figure is that the range where the force of law is needed is where the benefits from segregation are the largest.

The parameter α also matters for when discrimination is sustainable as an equilibrium and for the incentives to discriminate. One may think of an increase in α as "skill-biased technical change", but unless total factor productivity is adjusted when α changes this is technological regress: output falls when the task with the information problem becomes more important. Comparative statics with respect to α tend to be rather complicated and we will not describe the details. What is straightforward is that the larger is α, the less likely it is that assigning all B workers to the simple task is consistent with equilibrium. The effect is similar to an increase in λ^W: for large enough α even a worker that is very unlikely to be qualified is more productive in the complex task.

Effects on *incentives* to segregate for the dominant group are more involved. What seems robust is that incentives to segregate are small for very low and very high α. For low α the complex task is not very important, and exclusive rights to the task rights are not that valuable. For large α, the positive effects from making the input of labor in the simple task artificially high are small, so the loss from giving up the

[14] In the example, if $\lambda^W < 0.212$ then it is not possible in equilibrium to assign all B workers to the simple task.

exclusive rights to the complex task is negligible. In an intermediate range, the gain from the segregation regime is substantial for group W.[15]

The model has limitations as a model of the interplay between technology and incentives to segregate, but it does suggest that segregation of labor markets may unravel as a consequence of unskilled jobs becoming less important in the economy. We believe that some more refined version of this logic could potentially provide interesting economic explanations for the removal of government mandated discrimination, but much more work is needed for a serious analysis of such issues.

7. Summary and concluding remarks

The main contribution of this paper is to develop a model of statistical discrimination with true interaction effects between groups. Incentives to acquire human capital are affected not only by investment behavior in the own group, but also by human capital investments in the other group. This "cross-group effect" makes it possible for discrimination to arise also when there is a unique symmetric equilibrium. The dominant group is better off in equilibria with discrimination, which we view as an appealing property since this can rationalize why active measures are taken to institutionalize discrimination.

Our second contribution is that the model is a full-fledged general equilibrium model. This makes it possible to perform meaningful welfare analysis. Moreover, we believe that there are compelling reasons to avoid partial equilibrium models also for strictly positive analysis of economics of discrimination. Discrimination is usually considered as an economy-wide phenomenon and investigating the effects of different policies without allowing the wages to adjust may be to ignore the most important margins.[16]

Changes to the relative size of the discriminated group have intuitive effects in the model. In a parametric example we show that the larger is the discriminated group, the larger are the per capita gains from discrimination for the other workers. Furthermore, the larger is the discriminated group, the harder it is to sustain a discriminatory equilibrium. The model can therefore rationalize why a segregated labor market must be supported by coercive measures when the group in power is small.

Acknowledgments

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[15] In this intermediate range there are multiple local maxima of the gain. We do not fully interstand the reason for this.

[16] See Moro and Norman [12] for an analysis of Affirmative Action policies in this context.

Appendix A. Proofs

To conserve space we use $f_\pi(\theta)$ as shorthand notation for $\pi f_q(\theta) + (1-\pi)f_u(\theta)$ and $F_\pi(\theta)$ for $\pi F_q(\theta) + (1-\pi)F_u(\theta)$. Limits of integration are suppressed when integrating over the whole interval $[0, 1]$ and no confusion can arise. In the interest of brevity we have also omitted the proofs of some of the more intuitive intermediate steps. These are available in Moro and Norman [13].

A.1. Proof of Proposition 1

Proof (Sufficiency). Given $\pi \in (0, 1]$, let $\theta(\pi)$ solve the task assignment problem (3), $t : [0, 1] \to [0, 1]$ be the threshold rule with cutoff $\theta(\pi)$, and $(C(\pi), S(\pi)) = (\pi F_q(\theta), F_\pi(\theta))$ be the associated (aggregate) factor inputs. Suppose that each firm posts the wage schedule $w : [0, 1] \to R$ given by $w(\theta; \pi)$ in (5). Moreover, suppose (which is consistent with equilibrium) that *all* workers with signal realizations on the measurable set $\Theta_1 \subset [0, 1]$ break ties in favor of firm 1, while all workers on $\Theta_2 = \Theta\backslash[0, 1]$ break ties in favor of firm 2, where Θ_1 and Θ_2 are such that there exists $0 \leqslant \rho \leqslant 1$ such that

$$C_1(\pi) = \int_{\theta\in\Theta\cap[\theta(\pi),1]} \pi f_q(\theta)\, d\theta = \rho C(\pi),$$

$$S_1(\pi) = \int_{\theta\in\Theta_1\cap[0,\theta(\pi))} f_\pi(\theta)\, d\theta = \rho S(\pi). \tag{A.1}$$

This implies that $C_2(\pi) = (1-\rho)C(\pi)$ and $S_2(\pi) = (1-\rho)S(\pi)$. Given any ρ there is a multitude of sets Θ_1 and Θ_2 satisfying (A.1) and the profit for firm 1 is (firm 2 is symmetric)

$$\begin{aligned}
\Pi_1 &= y(C_1(\pi), S_1(\pi)) - \int_{\theta\in\Theta_1} w(\theta;\pi) f_\pi(\theta)\, d\theta \quad \text{from (5)} \\
&= y(C_1(\pi), S_1(\pi)) - \int_{\theta\in\Theta_1\cap[\theta(\pi),1]} \underbrace{\frac{\pi f_q(\theta)}{f_\pi(\theta)}}_{p(\theta,\pi)} \frac{\partial y(C(\pi), S(\pi))}{\partial C} f_\pi(\theta)\, d\theta \\
&\quad - \int_{\theta\in\Theta_1\cap[0,\theta(\pi))} \frac{\partial y(C(\pi), S(\pi))}{\partial S} f_\pi(\theta)\, d\theta \quad \text{from (A.1)} \\
&= y(C_1(\pi), S_1(\pi)) - \frac{\partial y(C(\pi), S(\pi))}{\partial C} \rho C(\pi) \\
&\quad - \frac{\partial y(C(\pi), S(\pi))}{\partial S} \rho S(\pi) = 0,
\end{aligned} \tag{A.2}$$

where the last equality uses homogeneity of degree zero of the partials of y and Euler's theorem. Suppose one firm deviates to $(w', t') \neq (w, t)$. Let C' and S' denote the implied factor inputs and let $a(\theta) \in [0, 1]$ denote the fraction of workers with signal θ that accepts a job at the deviating firm (tie-breaking rules are restricted so that a is integrable and independent of the investment, which is fine since we are arguing that the candidate wages and task assignments are supportable as an equilibrium). Since $w'(\theta; \pi) \geqslant w(\theta; \pi)$ for all θ such that $a(\theta) > 0$ the profit for the deviating firm, Π_i', satisfies

$$\Pi_i' \leqslant y(C', S') - \int w(\theta, \pi) a(\theta) f_\pi(\theta)\, d\theta, \tag{A.3}$$

where (here the assumption that ties are broken the same way by qualified and unqualified workers is used)

$$C' = \int t'(\theta) \pi f_q(\theta) a(\theta)\, d\theta \quad \text{and} \quad S' = \int (1 - t'(\theta)) a(\theta) f_\pi(\theta)\, d\theta. \tag{A.4}$$

Moreover $w(\theta, \pi) = \max\{p(\theta, \pi) y_1(C(\pi), S(\pi)), y_2(C(\pi), S(\pi))\}$, so

$$\begin{aligned}
&\int w(\theta, \pi) a(\theta) f_\pi(\theta)\, d\theta \\
&\quad = \int t'(\theta) w(\theta, \pi) a(\theta) f_\pi(\theta)\, d\theta + \int (1 - t'(\theta)) w(\theta, \pi) a(\theta) f_\pi(\theta)\, d\theta \\
&\quad \geqslant y_1(C(\pi), S(\pi)) \int t'(\theta) \pi f_q(\theta) a(\theta)\, d\theta + y_2(C(\pi), S(\pi)) \\
&\qquad \times \int (1 - t'(\theta)) a(\theta) f_\pi(\theta)\, d\theta \\
&\quad = y_1(C(\pi), S(\pi)) C' + y_2(C(\pi), S(\pi)) S'
\end{aligned} \tag{A.5}$$

implying that $\Pi_i' \leqslant y(C', S') - y_1(C(\pi), S(\pi)) C' + y_2(C(\pi), S(\pi)) S' \leqslant 0$ (by concavity and constant returns). $\square$

To prove necessity of the conditions in Proposition 1 we proceed by proving a sequence of intermediate results:

Lemma A.1. *Each firm earns a zero profit in any equilibrium.*

Proof. The proof, which is omitted, is based on the same style of reasoning as in the usual Bertrand competition model, but some work has to be done to make sure that the deviant firm attracts a distribution of workers such that efficiency in production is possible after the deviation (see [13] for the formal argument). $\square$

Lemma A.2. $w_1(\theta) = w_2(\theta)$ *for almost all* $\theta \in [0, 1]$ *in any equilibrium.*

Proof. The basic idea of the proof (available in [13]) is that if wages differ over a non-negligible set it is possible to attract the same workers at a lower cost. Since a deviation may trigger a change in the tie-breaking rules by the workers some work must be done to assure that productive efficiency does not decline. This is done by a deviation that attracts all workers. $\square$

Lemma A.3. $y(C_1, S_1) + y(C_2, S_1) = y(C(\pi), S(\pi))$.

Proof. By feasibility, $y(C_1, S_1) + y(C_2, S_1) \leqslant y(C(\pi), S(\pi))$, so assume for contradiction that $y(C(\pi), S(\pi)) - y(C_1, S_1) - y(C_2, S_1) = \delta > 0$. Suppose firm 1 offers $w_1'(\theta) = w_2(\theta) + \varepsilon$ for all θ and assigns all workers in accordance with a solution to (3). The implied profit is

$$\begin{aligned} \Pi_1'(\varepsilon) &= y(C(\pi), S(\pi)) - \int w_2(\theta) f_\pi(\theta)\, d\theta - \varepsilon \\ &> y(C_1, S_1) + y(C_2, S_2) - \int w_2(\theta) f_\pi(\theta)\, d\theta - \varepsilon. \end{aligned} \tag{A.6}$$

But (Lemma A.2) $w_1(\theta) = w_2(\theta)$ almost everywhere, so $\int_\theta w_2(\theta) f_\pi(\theta)\, d\theta$ is the sum of wages paid out by firms 1 and 2 before the deviation. By zero profits (Lemma A.1) this implies that $\Pi_1'(\varepsilon) = \delta - \varepsilon$, so for ϵ small enough the deviation is profitable. $\square$

Lemma A.4. *Suppose* $\langle w_1, w_2 \rangle$ *is a pair of equilibrium wage schedules and let* $\theta(\pi)$ *be the unique solution to* (3). *Then there is a pair* (k_s, k_c) *such that* (1) $w_i(\theta) = k_s$ *for* $i = 1, 2$ *and for almost all* $\theta < \theta(\pi)$, (2) $w_i(\theta) = p(\theta, \pi) k_c$ *for* $i = 1, 2$ *and for almost all* $\theta \geqslant \theta(\pi)$.

Proof. The two parts have almost identical proofs, so we prove only part (2), which may appear as less obvious. Let $w(\theta) = \max(w_1(\theta), w_2(\theta))$ and $(C(\pi), S(\pi))$ be the factor inputs corresponding to a solution to (3). For contradiction, suppose there is a set $A \subset [\theta(\pi), 1]$, where $m = \int_A \pi f_q(\theta)\, d\theta > 0$, and some $\delta > 0$ such that for all $\theta \in A$,

$$\begin{aligned} \frac{w(\theta)}{p(\theta,\pi)} &\leqslant \frac{1}{1 - F_q(\theta(\pi))} \int_{\theta(\pi)}^1 \frac{w(\theta)}{p(\theta,\pi)} f_q(\theta) - \delta \\ &= \frac{1}{\pi(1 - F_q(\theta(\pi)))} \int_{\theta(\pi)}^1 w(\theta) f_\pi(\theta) - \delta \\ &= \frac{1}{C(\pi)} \int_{\theta(\pi)}^1 w(\theta) f_\pi(\theta) - \delta. \end{aligned} \tag{A.7}$$

By continuity, there exists a set $B \in [0, \theta(\pi))$ such that $\int_B f_\pi(\theta)\, d\theta = \frac{S(\pi)}{C(\pi)} m$ and

$$w(\theta) \leqslant \frac{1}{F_\pi(\theta(\pi))} \int_{\theta(\pi)}^1 w(\theta) f_\pi(\theta) = \frac{1}{S(\pi)} \int_0^{\theta(\pi)} w(\theta) f_\pi(\theta) \tag{A.8}$$

for every $\theta \in B$. Consider a deviation by firm i, where it offers $w_i'(\theta) = w(\theta) + \varepsilon$ to workers with $\theta \in A \cup B$ and $w_i'(\theta) = 0$ for all other θ, and assigns workers from A to the complex task and workers from B to the simple task. The profit from this deviation is

$$
\begin{aligned}
\Pi' &= y\left(\int_{\theta\in A} \pi f_q(\theta)\, d\theta, \int_{\theta\in B} f_\pi(\theta)\, d\theta\right) \\
&\quad - \int_{\theta\in A\cup B} (w(\theta)+\varepsilon) f_\pi(\theta)\, d\theta \quad \text{(A.7) and (A.8)} \\
&\geqslant y(C(\pi), S(\pi))\frac{m}{C(\pi)} - \left(\frac{1}{C(\pi)}\int_{\theta(\pi)}^{1} w(\theta) f_\pi(\theta) - \delta\right)\int_{\theta\in A} \underbrace{p(\theta,\pi) f_\pi(\theta)}_{=\pi f_q(\theta)}\, d\theta \\
&\quad - \int_{\theta\in B} f_\pi(\theta)\, d\theta \left[\frac{1}{S(\pi)}\int_0^{\theta(\pi)} w(\theta) f_\pi(\theta)\right] \\
&\quad - \varepsilon \int_{\theta\in A\cup B} f_\pi(\theta)\, d\theta \quad \begin{array}{l} \int_A \pi f_q(\theta)\, d\theta = m \\ \int_B f_\pi(\theta)\, d\theta = \frac{S(\pi)}{C(\pi)} m \end{array} \\
&= y(C(\pi), S(\pi))\frac{m}{C(\pi)} - \left(\int_{\theta(\pi)}^{1} w(\theta) f_\pi(\theta) - \delta\right)\frac{m}{C(\pi)} \\
&\quad - \frac{S(\pi)}{C(\pi)} m \left[\frac{1}{S(\pi)}\int_0^{\theta(\pi)} w(\theta) f_\pi(\theta)\right] - \varepsilon \int_{\theta\in A\cup B} f_\pi(\theta)\, d\theta \\
&= \frac{m}{C(\pi)} \underbrace{\left(y(C(\pi), S(\pi)) - \int_{\theta\in[0,1]} w(\theta) f_\pi(\theta)\right)}_{=0 \text{ by Lemmas A1 and A3}} \\
&\quad + \delta \frac{m}{C(\pi)} - \varepsilon \int_{\theta\in\Theta'\cup\Theta''} f_\pi(\theta)\, d\theta.
\end{aligned} \tag{A.9}
$$

Hence, $\Pi' \geqslant \delta \frac{m}{C(\pi)} - \varepsilon \int_{\theta\in A\cup B} f_\pi(\theta)\, d\theta > 0$ for ϵ small enough, which together with Lemma A.2 establishes part (2) of the claim. The proof of the other half is symmetric. Removing the δ from (A.7) and inserting a δ in the inequality in (A.8) and again constructing A and B such that the factor ratio is as in a solution to (3) (i.e., satisfying the second condition in (A.8)), the rest of the argument is unaltered. □

Proof of Proposition 1 (Necessity). It remains to be shown that $k_s = y_2(C(\pi), S(\pi))$ and $k_c = y_1(C(\pi), S(\pi))$. Firm would make positive profits if $k_s < y_2(C(\pi), S(\pi))$ and $k_c < y_1(C(\pi), S(\pi))$ and negative profits if the inequalities go the other way. Hence, we need only consider the cases where the inequalities go opposite directions. The arguments are symmetric and we only consider the case with $k_s > y_2(C(\pi), S(\pi))$ and $k_c < y_1(C(\pi), S(\pi))$. If $\theta(\pi) = 0, (1)$ each firm makes a positive profits (loss), so the only case to consider is when $\theta(\pi)$ is interior. A necessary condition for optimality for problem (3) is that $y_1(C(\pi), S(\pi)) p(\theta(\pi), \pi) = y_2(C(\pi), S(\pi))$. Hence, there must be an interval $(\theta(\pi), \theta^*)$ where $w_i(\theta) = p(\theta, \pi) k_c < k_s$ for all $\theta \in (\theta(\pi), \theta^*)$. Consider the deviation

$$w_i'(\theta) = \begin{cases} w(\theta) + \varepsilon & \text{for } \theta \in (\theta(\pi), \theta^*), \\ 0 & \text{otherwise,} \end{cases} \qquad t_i'(\theta) = \begin{cases} 0 & \text{for } \theta \in (\theta(\pi), \theta'), \\ 1 & \text{for } \theta \in [\theta', \theta^*), \end{cases} \tag{A.10}$$

where θ' is set so that the factor ratio is as in the solution to (3) $\left(\frac{\int_{\theta'}^{\theta^*} \pi f_q(\theta)\, d\theta}{\int_{\theta(\pi)}^{\theta'} f_\pi(\theta)\, d\theta} = \frac{C(\pi)}{S(\pi)}\right)$. The profit is

$$\begin{aligned} \Pi' &= y\left(\int_{\theta'}^{\theta^*} \pi f_q(\theta)\, d\theta, \int_{\theta(\pi)}^{\theta'} f_\pi(\theta)\, d\theta\right) - \int_{\theta(\pi)}^{\theta'} w(\theta) f_\pi(\theta)\, d\theta - \int_{\theta'}^{\theta^*} w(\theta) f_\pi(\theta)\, d\theta \\ &> \frac{y(C(\pi), S(\pi)) \int_{\theta'}^{\theta^*} \pi f_q(\theta)\, d\theta}{C(\pi)} \\ &\quad - y_2(C(\pi), S(\pi)) \int_{\theta'}^{\theta^*} f_\pi(\theta)\, d\theta - y_1(C(\pi), S(\pi)) \int_{\theta'}^{\theta^*} \pi f_q(\theta)\, d\theta \\ &= \int_{\theta'}^{\theta^*} \pi f_q(\theta)\, d\theta [y(C(\pi), S(\pi)) - y_2(C(\pi), S(\pi)) S(\pi) \\ &\quad - y_1(C(\pi), S(\pi)) C(\pi)] = 0, \end{aligned} \tag{A.11}$$

which completes the proof of Proposition 1. □

A.2. Proof of Proposition 3

Proof. Suppose π' and π^* solve (17) and let $\pi' < \pi^*$. A calculation shows that $\int w^J(\theta; \pi^*) f_q(\theta) > \int w^J(\theta; \pi') f_q(\theta)$ and $\int w^J(\theta; \pi^*) f_u(\theta) > \int w^J(\theta; \pi') f_u(\theta)$, so workers with unchanged investment choices are strictly better off in the higher equilibrium. For workers with costs so that they choose to invest in the π^*-equilibrium, but not in the π'-equilibrium we have that

$$\begin{aligned} &\int w^J(\theta; \pi^*)(f_q(\theta) - f_u(\theta))\, d\theta \\ &\geqslant c > \int w^J(\theta; \pi')(f_q(\theta) - f_u(\theta))\, d\theta \\ &\Rightarrow \underbrace{\int w^J(\theta; \pi^*) f_q(\theta)\, d\theta - c}_{\text{payoff for agent } c \text{ in } \pi^* \text{ eq}} > \int w^J(\theta; \pi') f_q(\theta)\, d\theta \end{aligned}$$

$$+\int \underbrace{[w^J(\theta;\pi^*) - w^J(\theta;\pi')]}_{>0} f_u(\theta)\, d\theta$$
$$> \int w^J(\theta;\pi') f_q(\theta)\, d\theta > \underbrace{\int w^J(\theta;\pi') f_u(\theta)\, d\theta}_{\text{payoff for agent } c \text{ in } \pi' \text{ eq}} \tag{A.12}$$

so these workers are also strictly better off when $\pi = \pi^*$. $\square$

A.3. Proof of Proposition 4

The proof, which is relatively routine, establishes that $I(\pi)$ in (6) is continuous in π. Most work goes into establishing continuity at $\pi = 0$ and 1, see [13].

A.4. Proof of Proposition 5

Lemma A.5. $r(\boldsymbol{\pi})$ *is increasing in both arguments and strictly increasing in* π^J *for each* $\boldsymbol{\pi}$ *such that* $\theta^J(\boldsymbol{\pi}) > 0$.

The proof, which is omitted (available in [13]), uses the Kuhn–Tucker conditions to (8). The strategy is to assume that π^J increases and $r(\boldsymbol{\pi})$ decreases, which by use of the Kuhn–Tucker conditions implies that $\theta^J(\boldsymbol{\pi})$ must decrease for both groups. Using the definition of the equilibrium factor ratio, (19), this implies that $r(\boldsymbol{\pi})$ increases, a contradiction. $\square$

Lemma A.6. $\theta(\boldsymbol{\pi})$ *is continuously differentiable over the range where both thresholds are interior.*

Proof. The proof is a direct application of the implicit function theorem and omitted. $\square$

Proof of Proposition 5. To complete the proof for the weak version of the result, note that if $\pi^W < \pi^{W'}$ then Lemma A.5 implies that $r(\boldsymbol{\pi}) \leqslant r(\pi')$ and the difference in the gross benefits of invest for group B is

$$\Delta I^B = I^B(\boldsymbol{\pi}) - I^B(\boldsymbol{\pi}') = \int (w^B(\theta,\boldsymbol{\pi}) - w^B(\theta,\boldsymbol{\pi}'))(f_q(\theta) - f_u(\theta))\, d\theta. \tag{A.13}$$

The wage in the simple task increases and the wage in the complex task decreases for group B so $w^B(\theta,\boldsymbol{\pi}) - w^B(\theta,\boldsymbol{\pi}')$ is an increasing function of θ, which since F_q first order stochastically dominates F_u gives the result. For the strict part, in the case of a fully interior solution differentiate $I^B(\boldsymbol{\pi})$ (defined in (11) with respect to π^W to get

$$\frac{d}{d\pi^W} I^B(\boldsymbol{\pi}) = \underbrace{\frac{\partial^2 y(r(\boldsymbol{\pi}),1)}{\partial C \partial S}}_{>0} \underbrace{(F_q(\theta^B(\boldsymbol{\pi})) - F_u(\theta^B(\boldsymbol{\pi})))}_{<0 \text{ by MLRP}} \underbrace{\frac{\partial r(\boldsymbol{\pi})}{\partial \pi^W}}_{>0} + \underbrace{\frac{\partial^2 y(r(\boldsymbol{\pi}),1)}{\partial C^2}}_{<0} \int_{\theta^B(\boldsymbol{\pi})}^1 \underbrace{p(\theta,\pi^B)(f_q(\theta) - f_u(\theta))\, d\theta}_{>0 \text{ by MLRP}} \underbrace{\frac{\partial r(\boldsymbol{\pi})}{\partial \pi^W}}_{>0} < 0, \quad \text{(A.14)}$$

where $\frac{\partial^2 y(r(\boldsymbol{\pi}),1)}{\partial C \partial S} > 0$ and $\frac{\partial^2 y(r(\boldsymbol{\pi}),1)}{\partial C^2} < 0$ follows from strict quasi-concavity and $\frac{\partial r(\boldsymbol{\pi})}{\partial \pi^W} > 0$ by Lemma A.5. We leave to the reader to verify that if $\theta^W(\boldsymbol{\pi}) = 1$, we still have that $\frac{\partial r(\boldsymbol{\pi})}{\partial \pi^W} > 0$ if $\frac{\partial r(\boldsymbol{\pi})}{\partial \pi^W}$ is suitably reinterpreted as a right-hand derivative. Hence, (A.14) applies for this case as well. □

A.5. Proof of Proposition 6

Proof. Suppose $\pi^B = \psi$ and let $\pi^W(\psi)$ be the largest solution to $\pi^W = G_\psi(I^W(\pi^W, \pi^B = 0))$. By assumption there exists some π^* such that $\pi^* = G(I^W(\pi^*, \pi^*))$ and since $G_\psi(c) > G(c)$ for every c in the interior of the support and since (Proposition 5) $I^W(\pi^W, \pi^B)$ is decreasing in π^B it follows that $\pi^* = G(I^W(\pi^*, \pi^*)) < G(I^W(\pi^*, 0))$. This implies that $\pi^W(\psi) > \pi^*$ for every $\psi > 0$. For $(\pi^B, \pi^W) = (\psi, \pi^W(\psi))$ to be consistent with equilibrium it is sufficient that $\theta^B(\psi, \pi^W(\psi)) = 1$ (because $I^W(\pi^W, \pi^B) = I^W(\pi^W, 0)$ for all π^B such that $\theta^B(\psi, \pi^W) = 1$ this implies that $\pi^W(\psi)$ is a best response as well), which requires that

$$p(1,\psi) y_1(r(\psi, \pi^W(\psi)), 1) \leqslant y_2(r(\psi, \pi^W(\psi)), 1). \quad \text{(A.15)}$$

But $r(\psi, \pi^W(\psi)) \geqslant \lambda^W r^*$, where r^* is the factor ratio in the symmetric equilibrium π^* under distribution G. Hence, (A.15) is satisfied for ψ small enough, which establishes the claim. □

A.6. Proof of Proposition 7

Lemma A.7. *Given any $\pi \in [0,1]$ and $\pi^B \in [0,1]$, the average income in group W is always higher when investments are given (π^B, π) than when both groups have a fraction π of qualified workers.*

Proof. Consider a fictitious economy with one B agent and one W agent, where agents can either sell complex and simple labor at the market at prices $w_c = \frac{\partial y(*)}{\partial C}$ and $w_s = \frac{\partial y(*)}{\partial S}$ per effective unit, and where the $*$-argument is shorthand for evaluating the marginal products at equilibrium factor inputs given fractions of investors (π^B, π^W). Also assume that the fictitious representative J agent has available the technology y. The problem for a utility maximizing J-agent in this economy would then be to solve

$$\max_{C^J, S^J, \alpha^J, \beta^J} \quad y(\alpha^J C^J, \beta^J S^J) + w_c(1-\alpha^J)C^J + w_s(1-\beta^J)S^J,$$
$$\text{s.t.} \quad S^J \leqslant H(C^J) = \pi^J - C^J + (1-\pi^J)F_u\left(F_q^{-1}\left(\frac{\pi^J - C^J}{\pi^J}\right)\right). \tag{A.16}$$

From (A.16) one shows one solution is to set $\alpha^J = \beta^J = 0$ and provide exactly the effective factor inputs as in the equilibrium (all other solutions are equivalent in terms of total effective factor inputs. The indeterminacy that comes in is that there are positive α^J and β^J that can equalize marginal products in "domestic" production with market wages). Since "autarky" (the symmetric equilibrium) is a feasible solution to (A.16) for any (π^B, π^W) we conclude that group W is weakly better off when investments are (π^B, π) than if investments are (π, π) for any $\pi^B \in [0, 1]$. $\square$

Lemma A.8. *If π^* is the symmetric equilibrium and (π^{B*}, π^{W*}) is an equilibrium where $\pi^{B*} < \pi^{W*}$, then $\pi^{B*} < \pi^* < \pi^{W*}$.*

Proof. If π^* is the only symmetric equilibrium, then $\pi > G(I^J(\pi, \pi))$ for all $\pi > \pi^*$ and $J = B, W$ (otherwise there would be at least one additional equilibrium). If $\pi^* \leqslant \pi^{B*} < \pi^{W*}$, then Proposition 5 implies that $I^B(\pi^{B*}, \pi^{W*}) < I^B(\pi^{B*}, \pi^{B*})$ (the solution at (π^{B*}, π^{B*}) in a neighborhood is necessarily interior so that the inequality is strict). But, since $\pi^{B*} \geqslant G(I^B(\pi^{B*}, \pi^{B*})) > G(I^B(\pi^{B*}, \pi^{W*}))$ this contradicts the assumption that (π^{B*}, π^{W*}) is an equilibrium, so we conclude that $\pi^{B*} < \pi^*$. The proof of $\pi^* < \pi^{W*}$ is similar and left to the reader. $\square$

Proof of Proposition 7. Output increases if investments change from (π^*, π^*) to (π^{W*}, π^{W*}), and since groups are treated symmetrically and all output is paid back to workers the average wage also increases. By Lemma A.7, the average wage in group W is further increased when investments change from (π^{W*}, π^{W*}) to (π^{B*}, π^{W*}), which taken together with the first change means that the average wage in group W is higher in (π^{B*}, π^{W*}) than in (π^*, π^*). Finally, all agents in group W may choose (i.e., it is a feasible option) to invest exactly as in the symmetric equilibrium (π^*, π^*), in which case higher surplus follows immediately from the higher average wage. Since this is feasible, it must also be that investing at the higher rate must be weakly better for all agents who change their behavior across equilibria, so the average utility in group W must be higher in the equilibrium with discrimination. $\square$

A.7. Proof of Proposition 8

Proof (Part 1). By direct differentiation of (22)

$$\frac{d}{d\pi^W} Y(h(\pi^W), \pi^W) = \frac{\partial Y(h(\pi^W), \pi^W)}{\partial \pi^B} \frac{dh(\pi^W)}{d\pi^W} + \frac{\partial Y(h(\pi^W), \pi^W)}{\partial \pi^W}. \tag{A.17}$$

Appealing to the envelope theorem, the partial derivative of Y with respect to π^J is

$$\frac{\partial Y(\pi^B, \pi^W)}{\partial \pi^J} = \frac{\partial y}{\partial C}\lambda^J(1 - F_q(\theta^J)) + \frac{\partial y}{\partial S}\lambda^J(F_q(\theta^J) - F_u(\theta^J)), \tag{A.18}$$

where the arguments have been omitted for brevity. Since $dh(\pi^W)/\, d\pi^W = -\lambda^W/\lambda^B$ (A.17) and (A.18) can be combined to yield

$$\begin{aligned}\frac{d}{d\pi^W} Y(h(\pi^W), \pi^W) = \lambda^W \bigg(& \left(\frac{\partial y}{\partial C} - \frac{\partial y}{\partial S}\right)(F_q(\theta^B) - F_q(\theta^W)) \\ & + \frac{\partial y}{\partial S}(F_u(\theta^B) - F_u(\theta^W))\bigg).\end{aligned} \tag{A.19}$$

The Kuhn–Tucker conditions for an optimal solution to (8) may be written as

$$\begin{aligned} & -p(\theta^J(\boldsymbol{\pi}), \pi^J) \quad \frac{\partial y(r(\boldsymbol{\pi}), 1)}{\partial C} + \frac{\partial y(r(\boldsymbol{\pi}), 1)}{\partial S} + \gamma^J - \kappa^J = 0 \quad \text{for } J = B, W, \\ & \gamma^J \theta^J(\boldsymbol{\pi}) = 0, \qquad \kappa^J(1 - \theta^J(\boldsymbol{\pi})) = 0, \quad \gamma^J \geqslant 0, \quad \kappa^J \geqslant 0. \end{aligned} \tag{A.20}$$

Conditions (A.20) imply that if $\theta^B \leqslant \theta^W$, then $p(\theta^B, \pi^B) < p(\theta^W, \pi^W)$ and

$$0 < \frac{\partial y}{\partial C}(p(\theta^W, \pi^W) - p(\theta^B, \pi^B)) = \gamma^W - \gamma^B - \eta^W + \eta^B. \tag{A.21}$$

For (A.21) to hold, either γ^W or η^B is strictly positive. But, if $\gamma^W > 0$, then $\theta^W = 0 \Rightarrow \theta^B = 0$ (since $\theta^B \leqslant \theta^W$). By the Inada conditions this implies that $\frac{\partial y}{\partial S} = \infty$ and $\frac{\partial y}{\partial C} p(\theta^J, \pi^J) = 0$, violating (A.20). Next, $\eta^B > 0 \Rightarrow \theta^B = 1 \Rightarrow \theta^W = 1$ (since $\theta^B \leqslant \theta^{\mathrm{W}}$). By the Inada conditions this implies that $\frac{\partial y}{\partial S} = 0$ and $\frac{\partial y}{\partial C} p(\theta^J, \pi^J) = \infty$, again violating (A.20). We conclude that $\theta^B > \theta^W \Rightarrow F_q(\theta^B) - F_q(\theta^W) > 0$ and $F_u(\theta^B) - F_u(\theta^W) > 0$. Finally, we observe that this also implies that $\theta^W < 1 \Rightarrow \eta^J = 0 \Rightarrow \frac{\partial y}{\partial C} p(\theta^J, \pi^J) \geqslant \frac{\partial y}{\partial S}$, which since $p(\theta^J, \pi^J) < 1$ guarantees that $\frac{\partial y_1}{\partial C} > \frac{\partial y_1}{\partial S}$. Thus, all terms in (A.19) are strictly positive, establishing the first part. $\square$

Proof (Part 2). Again using the chain rule and that $dh(\pi^W)/\,d\pi^W = -\lambda^W/\lambda^B$

$$\begin{aligned}
&\frac{d}{d\pi^W}\left(\lambda^B \int_{\underline{c}}^{G^{-1}(h(\pi^W))} cg(c)\,dc + \lambda^W \int_{\underline{c}}^{G^{-1}(\pi^W)} cg(c)\,dc\right)\\
&\quad = -\lambda^W G^{-1}(h(\pi^W))g(G^{-1}(h(\pi^W)))\,\frac{dG^{-1}(h(\pi^W))}{d\pi^B}\frac{dh(\pi^W)}{d\pi^W}\\
&\qquad +\ \lambda^W G^{-1}(\pi^W)g(G^{-1}(\pi^W))\,\frac{dG^{-1}(\pi^W)}{d\pi^W}\\
&\quad = \ \lambda^W(G^{-1}(\pi^W) - G^{-1}(\pi^B)) > 0, \qquad\qquad (A.22)
\end{aligned}$$

where the last equality follows since $g(G^{-1}(\pi^J))\frac{dG^{-1}(\pi^J)}{d\pi^J} = 1$ by the inverse function theorem and the inequality follows since G^{-1} is strictly increasing. □

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