The Performance of the Pivotal Voter Model in Small Scale Elections: Evidence from Texas Liquor Referenda

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Question

- How well does the pivotal voter model of electoral turnout perform in small scale elections ?

- Ledyard (1984): rational voters motivated by the chance they might swing the election in a strategic environment + incomplete information

- We estimate the parameters of the model using data from Texas liquor liberalization referenda

Motivation

- Palfrey and Rosenthal (1985): as the number of eligible voters goes to infinity, only those with negative or zero cost vote

- This results is often used to dismiss the model as a reasonable explanation of voter turnout in large elections (Green and Shapiro 1984, Feddersen 2004)

- However this does not mean that it is in not a good model in small scale elections

Other theories of voter turnout

- The Group-based models : groups coordinate their turnout
  - "Ethical" models (Feddersen and Sandroni 2002): everybody follows the rule maximizing the groups' aggregate payoff
  - "Mobilization" models (Shachar and Nalebuff): leaders organize followers

- Expressive voting theories
  - The intensity model: voters are more likely to vote if they feel more strongly about the issue

Empirical regularities regarding turnout

- Strong relationship to sociodemographic variables (Ashenfelter and Kelly 1975, Rosenstone 1980)

- Likelihood of being decisive: conflicting evidence
  No: Ashenfelter and Kelly (1975);
  Yes: Silberman and Durden (1975), Rosenthal and Sen (1973)

- More direct test: Hansen, Palfrey and Rosenthal (1987) assume a symmetric pivotal model and use only "close" elections.

- Experimental approach: Levine and Palfrey (2005)

- Technical difficulties + Hard to find the right data

The data we have

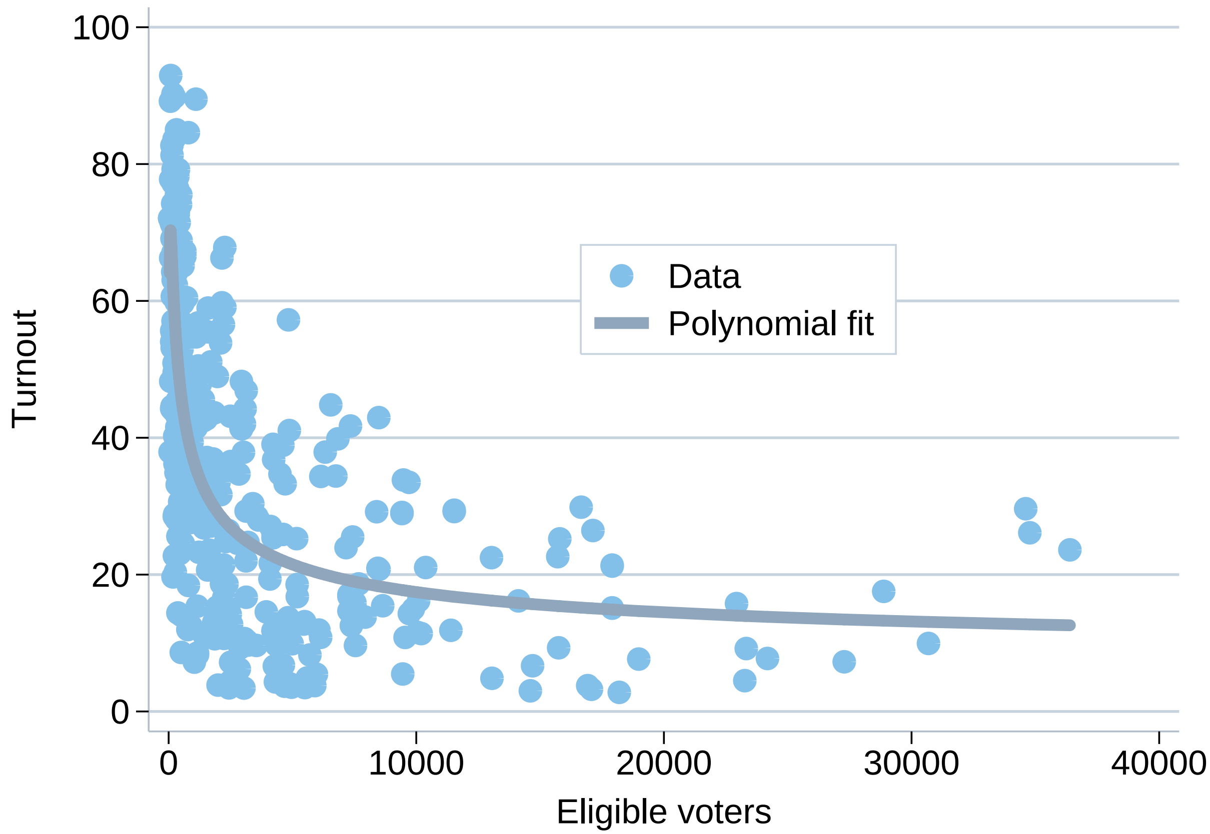
- Coate and Conlin (2004) assembled data on 366 local liquor referenda in Texas between 1976 and 1996.
  Prior to the referendum the local jurisdiction prohibited the sale of alcohol

- Until 2001 liquor referenda were held on special dates, different than standard election days

- Additional information about jurisdictions from the Census
  (more details later)

Data - turnout

Voter turnout as a percent of eligible voters: $(Yes + No) / Eligible$

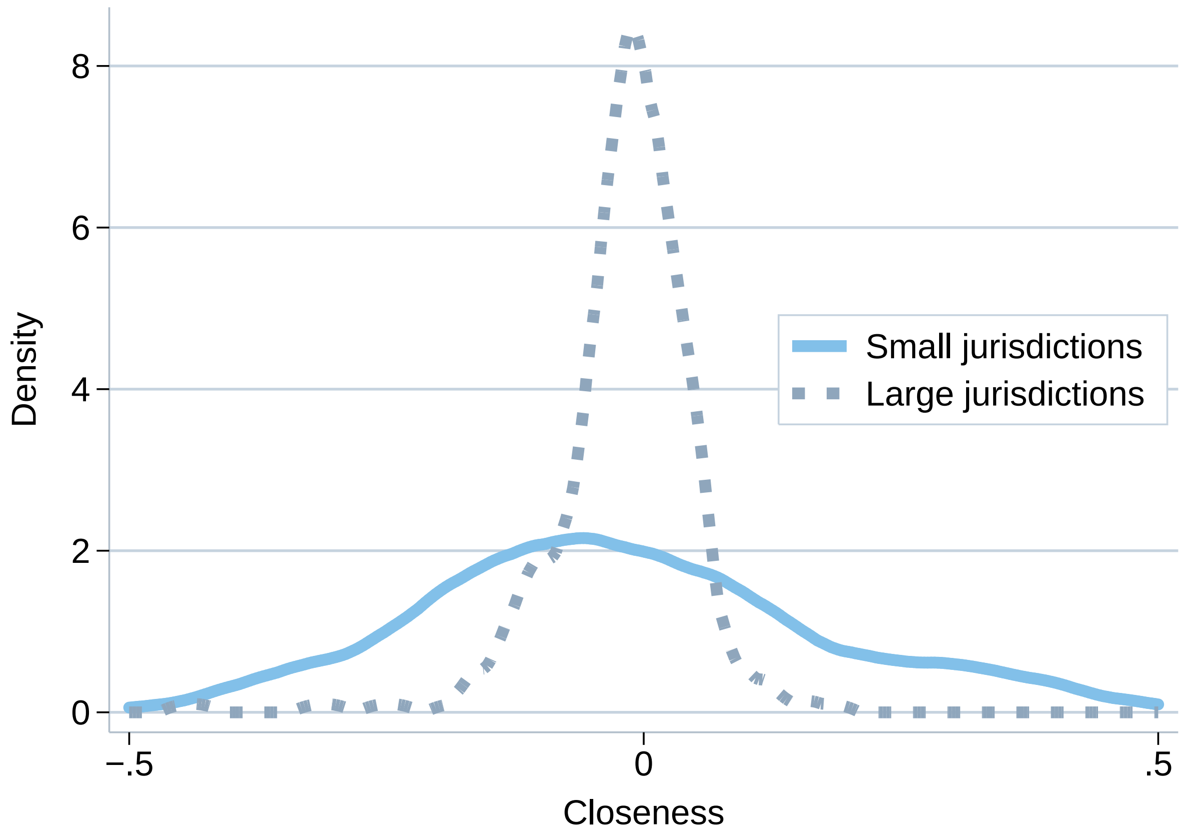


Data - turnout

	Eligible voters n	N. of obs.	Perc. turnout
"Small" jurisdictions	$n < 247$	48	0.62
	$247 < n < 434$	48	0.55
	$434 < n < 900$	48	0.43
"Large" jurisdictions	$900 < n < 2245$	72	0.32
	$2245 < n < 5170$	72	0.23
	$5189 < n < 30000$	72	0.18
	$n > 30000$	6	0.26

Data - closeness

Percent vote difference: $(Yes - No)/(Yes + No)$



The pivotal voter model

- Citizens, indexed by $i \in \{1, ..., n\}$ vote to relax liquor restrictions
- μ : Probability citizen i is a supporter
- b : Supporters' willingness to pay for the relaxation
- x : Opposers' willingness to pay to avoid the relaxation
- $c_i \sim U[0, c]$: Cost of voting for citizen i
- Each citizen knows her cost, but only knows the distribution of costs of the other citizens

The pivotal voter model (cont.)

- Strategy: $f : [0, c] \times \{\text{supporter,opposer}\} \longrightarrow \{\text{vote,abstain}\}$

- Focus on symmetric equilibria, where all supporters and opposers use the same strategy

- W.l.o.g assume they use a "cutoff" strategy:

$$\begin{aligned} \text{supporter } i \text{ votes if } c_i &\leq \gamma_s^* \\ \text{opposer } i \text{ votes if } c_i &\leq \gamma_o^* \end{aligned}$$

The probability of an election outcome

- $P(s)$: probability that s of the other $n-1$ voters are supporters

$$P(s) = \binom{n-1}{s} \mu^s (1-\mu)^{n-1-s}$$

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- $\rho(Yes, No; \gamma_s^*, \gamma_o^*)$: probability Yes supporters vote, and No opposers vote

$$= \sum_{s=Yes}^{n-1-No} P(s) \quad \binom{s}{Yes} \left(\frac{\gamma_s^*}{c}\right)^{Yes} \left(1 - \frac{\gamma_s^*}{c}\right)^{s-Yes}$$
$$\binom{n-1-s}{No} \left(\frac{\gamma_o^*}{c}\right)^{No} \left(1 - \frac{\gamma_o^*}{c}\right)^{n-1-s-No}$$

(Go to: likelihood function)

Equilibrium conditions

Assume n even, supporters win when outcome is tied

$$\overbrace{\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s^*, \gamma_o^*) \cdot b}^{\text{Expected benefit to a supporter}} = \gamma_s^*$$

$$\overbrace{\sum_{v=0}^{n/2-1} \rho(v, v; \gamma_s^*, \gamma_o^*) \cdot x}^{\text{Expected benefit to an opposer}} = \gamma_o^*$$

The data

- 366 local liquor elections in Texas between 1976 and 1996 where prior to the election the voting jurisdictions prohibited the retail sale of all alcohol.

Jurisdiction	N	Voters	Supporters win	Close elections*
Small	144	< 900	65	28
Large	222	> 900	87	64

* $< 10\%$ margin of victory

- Additional information from the U.S. Census and Churches & Church Membership in the U.S.

The data: additional info

	Small jurisdictions	Large jurisdictions
Number of referenda	144	222
Jurisdiction characteristics		
Voting age population	370 (200)	6,539 (8,742)
Fraction of baptists	52% (11)	46% (14)
Located in an MSA	44% (50)	43% (50)
Incorporated city or town	95% (22)	42% (50)
Referendum characteristics		
Beer/wine	46% (50)	37% (48)
Off-premise	40% (49)	39% (49)
Off- and on-premise	15% (35)	24% (43)
More liberal than county	42% (49)	28% (45)
Held on weekend	68% (47)	72% (45)

Paper: Table 1, page 10

Identification

4 parameters: b, x, μ, c

- Only relative prices matter $c = 1$
- The magnitude of b, x affect turnout
- $b - x$ and μ are separately identified because their effect varies with the size of the jurisdiction
  - e.g. when turnout is high, the vote share is close to μ, the fraction of supporters, and $b - x$ has not much effect
  - when turnout is low, then both μ and $b - x$ affect the vote share.

Estimation

- For each jurisdiction j, we assume:

$$\begin{aligned}
\text{supporter's benefit} \qquad b_j &= \exp\left(\boldsymbol{\beta}^b \cdot \mathbf{z}_j^b\right) \\
\text{opposer's benefit} \qquad x_j &= \exp\left(\boldsymbol{\beta}^x \cdot \mathbf{z}_j^x\right) \\
\text{fraction of supporters} \qquad \mu_j &= \frac{\exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)}{1 + \exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)} \\
\text{cost distribution upper bound} \qquad c_j &= \exp\left(\beta^c \cdot z_j^c\right)
\end{aligned}$$

- Variables used:
  $\mathbf{z}^b, \mathbf{z}^x = 1$, off-premise, off/on-premise, city, more liberal than cty.
  $\mathbf{z}^\mu = 1$, fraction of baptists, MSA
  $z^c =$ election on weekend (c normalized)

The likelihood

- observables $\mathbf{z}_j$ determine b_j, x_j, μ_j, c_j for each jurisdiction j

- The equilibrium conditions determine a set of M_j equilibria

- Use an (arbitrary) equilibrium selection rule
  denote the selected equilibrium $(\gamma^*_{sj}, \gamma^*_{oj})$.

- Likelihood of observing an outcome
  conditional on equilibrium thresholds $(\gamma^{m*}_{sj}, \gamma^{m*}_{oj})$

$$L(\Omega) = \prod_j \rho(Yes_j, No_j; \gamma^*_{sj}, \gamma^*_{oj})$$

(Go to: ro function)

Results: parameters

Parameter/Variable (ln L : -5694.21)		Estimate	Marg. Eff.
μ:	Fraction of baptists	-0.058 (0.188)	-0.015
	Located in an MSA	-0.089 (0.072)	-0.022
	Constant	0.062 (0.097)	
b:	Off-premise	0.182 (0.086)	2.85
	Off- and on-premise	-0.642 (0.232)	-7.89
	Incorporated city or town	1.819 (0.354)	13.68
	More liberal than county	0.199 (0.068)	3.15
	Constant	0.875 (0.405)	
x:	Off-premise consumption	0.097 (0.082)	1.56
	Off- and on-premise	-0.589 (0.253)	-7.58
	Incorporated city or town	1.791 (0.340)	13.97
	More liberal than county	0.361 (0.062)	5.90
	Constant	0.886 (0.370)	
c:	Held on weekend	-0.172 (0.085)	-0.16

Paper: Table 2, page 17

Results: mean estimates

Parameter	Mean estimate
Fraction of supporters μ	0.500 (0.011)
Supporters' benefit b	15.52 (4.81)
Opposers' benefit x	15.90 (5.12)
Upper bound on cost c	0.892 (0.074)
Supporters that vote $\frac{\gamma_s}{c}$	0.516 (0.167)
Opposers that vote $\frac{\gamma_o}{c}$	0.530 (0.174)

An average voting cost $(c/2)$ of \$10 implies $b =$ \$348 and $x =$ \$357

Multiplicity of equilibria not salient.

Paper: Table 3, page 18

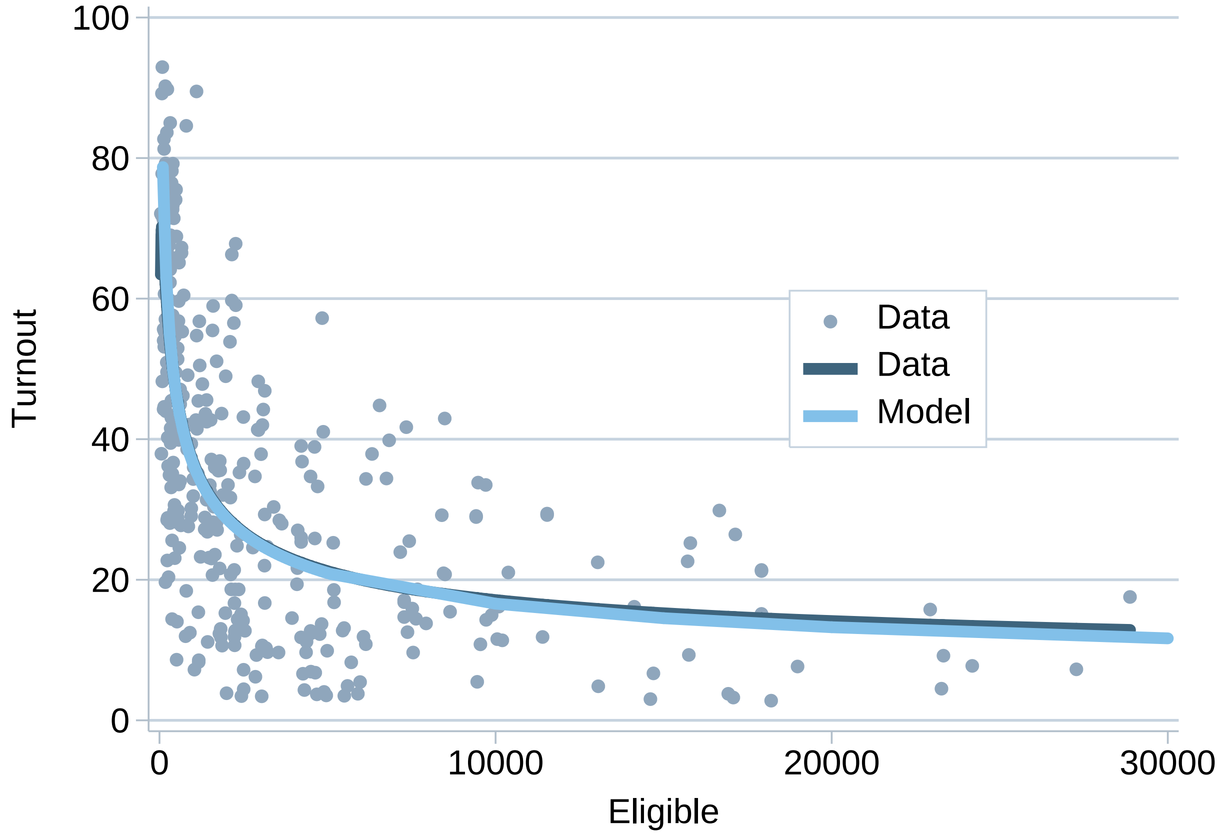
Goodness of fit, turnout

Eligible voters n	N. of obs.	Data	Pivotal-voter model
$n < 247$	48	0.62	0.65
$247 < n < 434$	48	0.55	0.51
$434 < n < 900$	48	0.43	0.40
All $n < 900$	144	0.54	0.52
$900 < n < 2245$	72	0.32	0.19
$2245 < n < 5170$	72	0.23	0.11
$5189 < n < 30000$	72	0.18	0.08
$n > 30000$	6	0.26	0.06

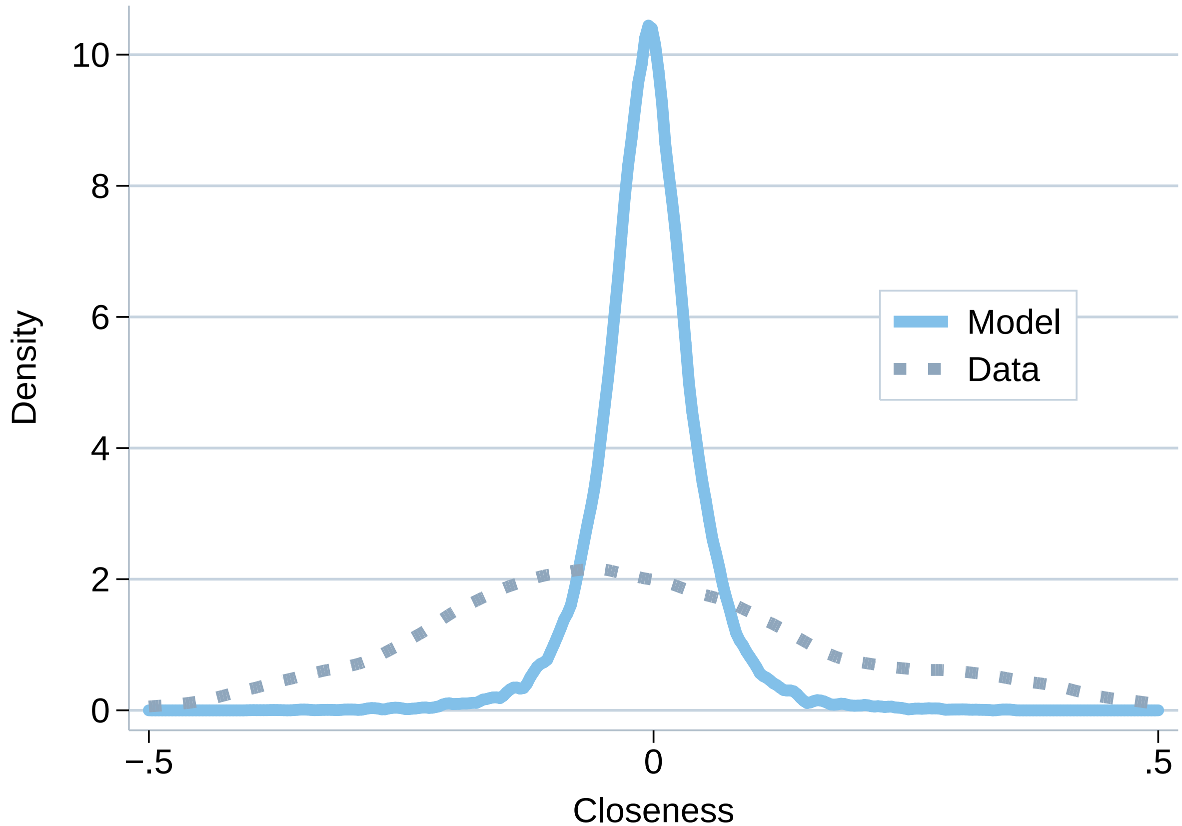
Paper: first part from Table 4, page 19

Turnout, mean estimates

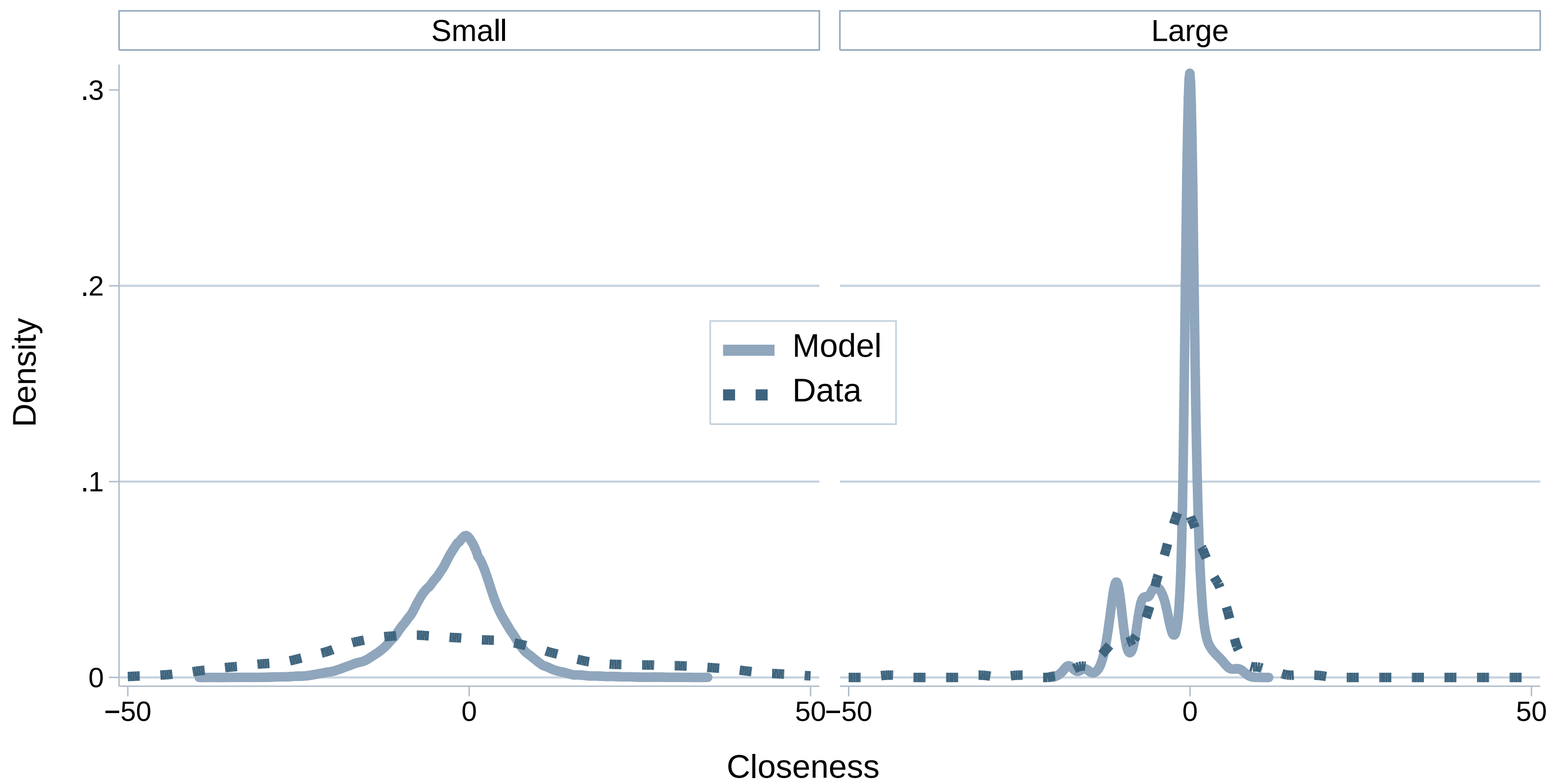
The model is, in principle, capable of generating “high” turnout



Goodness of fit, closeness



Goodness of fit, closeness by size



The intensity model

- α = strength of voters' desire for policy.

- As before: cost $c_i \sim U[0, c]$,
  fraction of supporters μ,
  benefit to supp. b,
  benefit to opp. x.

- Voter i votes if

$$\begin{aligned} c_i &\leq \alpha b \\ c_i &\leq \alpha x \end{aligned}$$

Note: α constant in size (makes it more difficult to match data)

Intensity model, parameter estimates

Parameter / Variable (ln L: -4567.0)		Estimate	Marginal Effect
μ:	Fraction of baptists	-0.076 (0.117)	-0.0001
	Located in an MSA	-0.038 (0.024)	-0.074
	Constant	-0.798 (0.076)	
αb:	Off-premise	0.133 (0.019)	0.115
	Off- and on-premise	-0.213 (0.032)	-0.170
	Incorporated city or town	0.612 (0.040)	0.404
	More liberal than county	0.036 (0.019)	0.031
	Constant	-0.780 (0.051)	
αx:	Off-premise	0.055 (0.015)	0.021
	Off- and on-premise	-0.581 (0.032)	-0.177
	Incorporated city or town	0.219 (0.285)	0.075
	More liberal than county	0.296 (0.015)	0.113
	Constant	-1.277 (0.033)	
c:	Held on weekend	0.027 (0.012)	0.028

Paper: Table 5, page 22

Intensity model, mean estimates

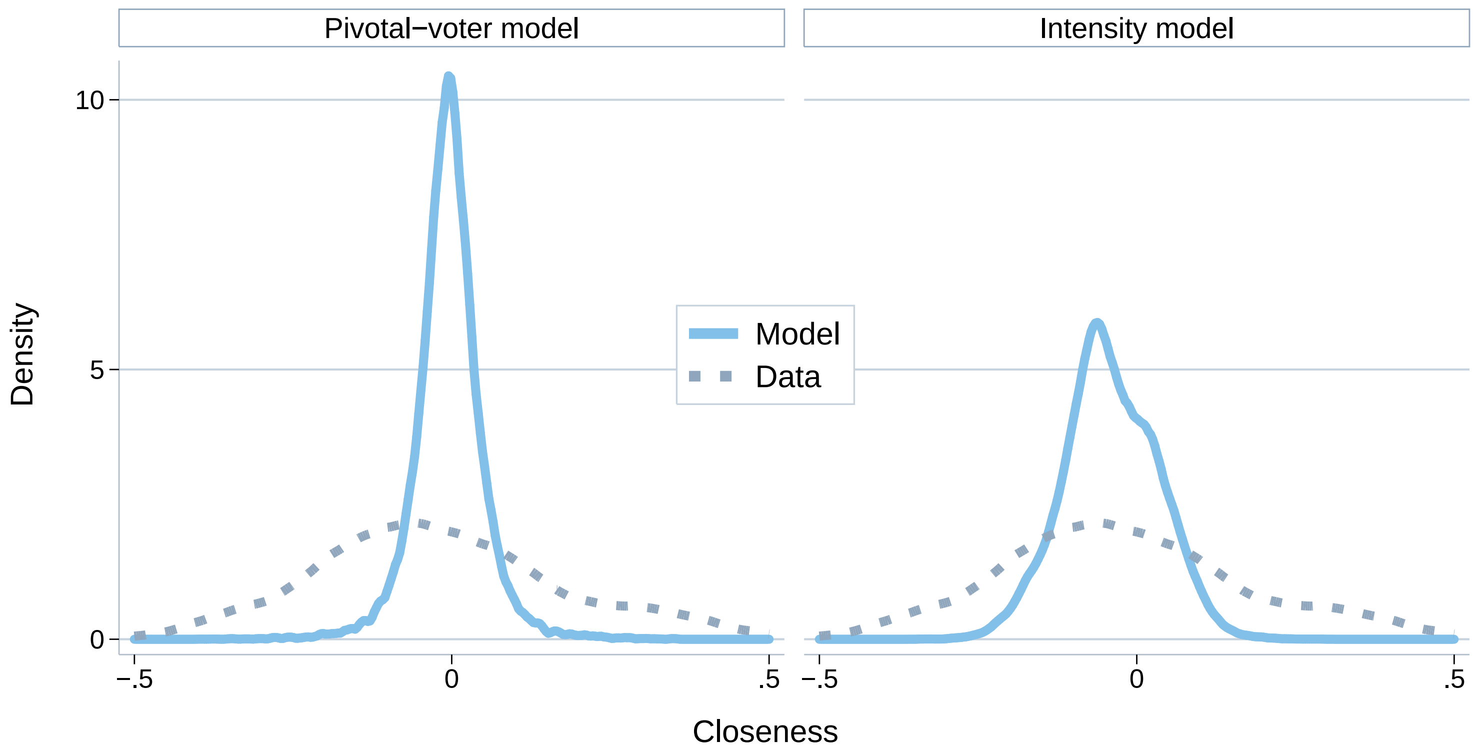
Parameter	Mean estimate
Fraction of supporters μ	0.423 (0.043)
Supporters' expressive benefit αb	0.585 (0.138)
Opposers' expressive benefit αx	0.504 (0.137)
Upper bound on cost c	1.005 (0.003)
Supporters that vote $\frac{\gamma_s}{c}$	0.583 (0.138)
Opposers that vote $\frac{\gamma_o}{c}$	0.501 (0.137)

Intensity model, goodness of fit, turnout

Eligible voters n	N. of Obs.	Data	Intensity model
$n < 247$	48	0.62	0.50
$247 < n < 434$	48	0.55	0.52
$434 < n < 900$	48	0.43	0.49
All ($n < 900$)	144	0.54	0.50

Paper: first part from Table 7, page 24

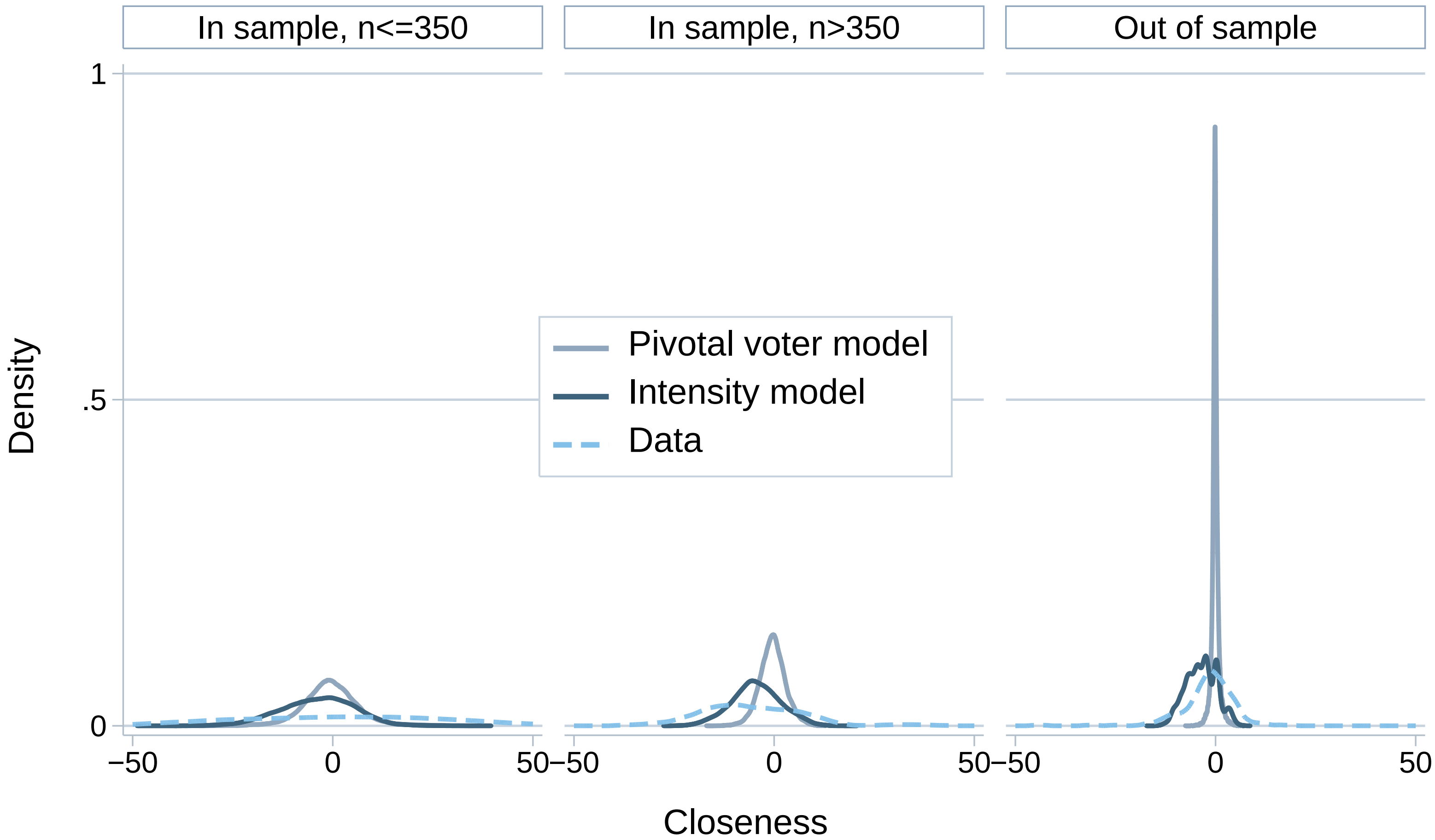
Closeness, comparison between models



A Vuong non-nested models test of the null hypothesis that the two models are equally close to the true dgp does not reject the null 0 - it does if we make α depend on size)

Paper: see Figure 4, page 25

Closeness, comparison, by size



Conclusion

- The pivotal voter model seems to be able to perform well in predicting turnout

- It does not perform well in predicting closeness of the election

- A simple model based on expressive voting does better

- The dependency of turnout on size does not necessarily depend on the strategic nature of the voting choice.