Endogenous Inequality in a Trade Model with Private Information[*]

Andrea Moro[§] Peter Norman[¶]

December 8, 2000

Abstract

We develop a general equilibrium model of trade between identical countries. The model is similar to a $2 \times 2 \times 2$ Heckser-Ohlin model, but the factors of production, skilled and unskilled labor, are endogenously determined from human capital investments by the workers. Since firms are only able to observe human capital investments with noise, an informational externality arises. This externality combined with general equilibrium price effects makes incentives to invest a function of investment behavior both in the home and the foreign country. We show that there may be equilibria when countries specialize as rich, high-tech countries and poor low-tech countries respectively, also when the basic autharky model has a unique equilibrium. There are potential efficiency gains from specialization. Protectionism *may* make the poor country better off, but we can construct examples where the efficiency gains are large enough to make the specialization equilibrium better than the unique autharky equilibrium also for the poor country.

[*]We thank Hal Cole, Rody Manuelli, Andrew Postlewaite, Robert Staiger, Kjetil Storesletten, Scott Taylor and Fabrizio Zilibotti for comments, suggestions and discussions. We have also benefited from feedback from seminar participants at several conferences and institutions. Support from NSF Grants #SES-0003520 and #SES-0001717 is gratefully acknowledged. The research for this paper was done partly when Peter Norman was visiting IIES Stockholm and he is grateful for their hospitality.

[§]Department of Economics, University of Minnesota. Email: amoro@atlas.socsci.umn.edu

[¶]Department of Economics, University of Wisconsin-Madison. Email: Pnorman@facstaff.wisc.edu

1 Introduction

It is widely agreed that standard neoclassical growth and international trade models are inadequate tools for understanding the huge differences in economic development across countries and regions[1]. This failure of traditional theory has motivated a great deal of recent work. Some of it emphasizes institutions and seeks to explain the differences by politically induced distortions (Parente and Prescott [13], [14]), but the most common departure from the traditional models is to introduce some externality (Footnote 1 contains a few examples), which is typically rationalized as a reduced form for some network externality or knowledge spillovers.

The idea that knowledge and human capital may be important factors is intuitively appealing and, while causality is an issue, it is well documented that variables that we think of as proxies for human capital are correlated with measures of economic development (see for example Benhabib and Spiegel [2], Psacharopoulos [15] or Topel [19]). But, most models in the literature takes a reduced form approach and introduce external effects as a primitive assumption and we think that this black-box-approach should be a reason for some concern. While there may be answers to questions like "why doesn't knowledge travel across national borders?" or "why can't expertise be sold to the nations that need it the most?" it is hard to evaluate the models when these explanations are left out of the analysis[2].

In this paper we analyze a model where an externality is *derived* rather than taken as a primitive of the model. Here it should be pointed out that the "new theory of economic geography" also analyzes derived externalities, so our model is not the first to achieve this. These models combine scale economics, imperfect competition, trade frictions, which together with mobile labor leads to "pecuniary" externalities.

What is novel in our analysis is that we generate endogenous externalities in a model with competitive markets. This is accomplished by introducing an arguably rather reasonable informational

[1] See Lucas [8], [9] and Romer [18] for elaborations in the context of neoclassical growth theory. In the trade literature, Puga and Venables [17] offers a succinct statement of the shortcomings of traditional trade models for addressing development issues. The current interest in applying new models of economic geography and agglomeration (Krugman [6], Krugman and Venables [7], Puga and Venables [16] and others) in studies of cross country income differentials would also be hard to understand if the traditional theory would be viewed as an adequate tool.

[2] This concern has been raised before in different guises. See the discussion about local versus global external effects in Lucas [9] and Krugman [6].

asymmetry, where it is assumed that workers know more about their capabilities than the firms do. Combined with endogenous human capital acquisition this leads to an informational externality that creates incentives for countries to specialize in our model.

Apart from the private information we want to deviate as little as possible from textbook trade theory. We assume that there are two countries, two factors of production, two goods, a competitive market, constant returns production functions and no trade frictions. Indeed, if human capital is exogenously fixed our model basically reduces to the Hecksher-Ohlin-Samuelson model.

The departure from standard theory is that factors of production are endogenously determined through human capital investments. We ignore capital completely and assume that the relevant inputs in production are workers with different skills. Specifically, we assume that there is a *complex job* and an *simple job* and each worker faces a binary investment decision in a job specific skill. In order to be productive in the complex job a worker must undertake a costly investment in human capital, while all workers are equally productive in the simple job. Firms in each sector have available a constant returns technology that transforms the labor inputs to a consumption good. One sector, the "high-tech" sector, is more intensive in complex labor than the other sector.

Both countries have access to the same technology, have identical distributions of investment costs, and preferences over the two goods are identical and homothetic, so the model can not generate any trade or income inequality if the human capital investments are perfectly observable. Factor price equalization would then guarantee that incentives to invest are the same in each country and any equilibrium must therefore be symmetric and without trade.

In our model, trade and inequalities are possible only because an informational asymmetry. Firms cannot observe whether the worker is qualified, only a noisy signal. One interpretation is that the signal represents the curriculum vitae of a worker, which contains *imperfect* information about the productive characteristics of the worker.

Firms act competitively and pay workers the value of their expected marginal products, which depend on observable characteristics, and also on knowledge about aggregate investment behavior in the economy. That is, a direct consequence of Bayesian updating is that investments in the population as a whole affects the probability assessment that any particular worker is qualified. The asymmetric information thus creates an *informational externality,* making the incentives to invest in human capital dependent on aggregate investments.

The informational externality *may* generate multiple equilibria in the autarky model, in which case equilibria with trade and inequality would be expected, but hard to interpret. However, the autarky model may also have a unique equilibrium (for the parametric class we use for our examples this is always the case) and countries may still specialize in equilibrium under free trade. We focus on this possibility.

Trade and inequalities are driven by the interplay between the informational externality and general equilibrium price effects. The latter make incentives to invest depend on aggregate investment behavior also in the other country. The higher are investments in the other country, the less valuable are workers with human capital in both countries, so an increase in investments abroad narrows the wage differential between the high and the low skilled job at home.

This type of price effects are present also with perfect information, but wages would then depend only on the investment decision, while nationality would be irrelevant. With perfect information there would therefore be a unique equilibrium without trade. Asymmetric information changes this because the informational externality creates a "direct effect" on wages in the country where investments change. This interplay between price effects and the local informational externality makes it possible for identical countries to specialize as rich countries exporting high tech goods and poor countries producing low tech goods, also if the autarky equilibrium is unique.

The country specializing as a low skill country is poorer and worse off than the other country. Still, nothing can be said about how the specialization equilibrium compares with the autarky equilibrium for the poor country. We construct an example where the poor country is better off than in the unique autarky equilibrium and another example where it goes the other way.

On the production side, specialization increases the production possibilities in the world economy, due to fewer mistakes in task assignments when countries specialize. The downside is that low cost investors are replaced by investors with higher investment costs when the economy moves towards increased specialization. Taken together this means that specialization has an ambiguous net effect on economic efficiency and examples can be constructed going either way.

A feature of our model that deserves to be emphasized is that workers in a country that is poor in human capital in equilibrium have worse incentives to invest in human capital than the country that is rich in human capital. In models of perfect information this property is hard to obtain and if countries are poor because of a lack of human capital one wonders why these countries do not invest more in human capital. This is usually dealt with either by assuming some exogenous differences[3] or by assuming that high human capital countries (or high knowledge countries) have a comparative advantage in producing more human capital (knowledge).

We also note that, unlike more reduced form models that take knowledge spillovers as a primitive, the informational externality *must* be local. Since countries are defined in terms of barriers to labor mobility and local firms only care about workers on the local labor market, asymmetric information creates a purely local externality.

2 The Model

Our model combines elements from our previous work on statistical discrimination with a traditional trade setup. We take the informational technology and model of human capital accumulation from our model of discrimination in Moro and Norman [10]. The rest of the model is set up to be as close to a standard $2 \times 2 \times 2$ trade model as possible.

2.1 Human Capital Investments

There are two countries indexed by $j = h, f$. Each country has a continuum of workers with heterogenous costs of investment in human capital. Agents are distributed on $[\underline{k}, \overline{k}]$ according to a continuous and strictly increasing distribution function G, where $\underline{k} \leq 0$ and $\overline{k} > 0$[4]. Prior to entering the market each agent $k \in [\underline{k}, \overline{k}]$ has to choose between investing or not investing in human capital. Agents who invests incur utility cost k, while agents who don't invest incur no cost.

2.2 Information Technology

After the investments, nature assigns each worker a signal $\theta \in \Theta$ (see Section 2.5 for interpretations). In Section 5 we assume that Θ is discrete, but in general it is more tractable to let $\Theta = [0, 1]$. For the continuous version we assume that θ is distributed according to density f_q if the worker invested and f_u otherwise. The densities f_q and f_u are continuously differentiable, bounded away from zero and satisfy the strict monotone likelihood ratio property $f_q(\theta)/f_u(\theta) < f_q(\theta')/f_u(\theta')$ for all θ, θ'

[3] See, for example, Acemoglu and Zilibotti [1]

[4] The rationale for allowing $\underline{k} < 0$ is that even the slightest mass of workers who derives a positive utility out of the investment eliminates the possibility of a trivial equilibrium in the autharky model.

such that $\theta < \theta'$. This implies that qualified workers are more likely to get higher values of θ than unqualified workers. We let F_q and F_u denote the associated cumulative distributions.

2.3 Production Technology

There are two consumption goods, x_1 and x_2, both produced solely from labor input in two different jobs. We refer to these jobs as *the complex task* and *the simple task.* Call workers who invested in the human capital *qualified* workers and workers who did not *unqualified.* Unqualified workers employed in the complex task do not contribute at all to output, so the effective input of complex labor in industry i, c_i, is the quantity qualified workers employed in the complex task. In the simple task on the other hand human capital is not needed, so the effective input of simple labor in industry i, s_i, is simply the number of workers (qualified and unqualified) employed in this task.

It is crucial that human capital investments affect productivity asymmetrically in the two jobs. However, the extreme assumptions that non-investors are totally useless in the complex job and that the investment does not improve productivity at all in the simple job are only for expositional simplicity.

Given inputs c_i and s_i the output in industry i is $y^i(c_i, s_i)$ where $y^i : R_+^2 \to R_+$ is a continuously differentiable neoclassical production function, satisfying constant returns to scale. To rule out "factor intensity reversals" we assume that,

A1 $\frac{\frac{\partial y^1(c,s)}{\partial c}}{\frac{\partial y^1(c,s)}{\partial s}} > \frac{\frac{\partial y^2(c,s)}{\partial c}}{\frac{\partial y^2(c,s)}{\partial s}}$ for all $c, s > 0$.

This single crossing condition on isoquants says that the increase needed in complex labor to keep output constant after a decrease in the input of simple labor is smaller in sector one (given a common factor ratio). Again, the example in Section 5 is slightly different: there $y^1(c_1, s_1) = c_1$ and $y^2(c_2, s_2) = s_2$, which can be viewed as a limiting case of a technology satisfying **A1.**

2.4 Preferences

The agents in the model care about consumption and investment costs. Preferences over consumption bundles (given investment behavior) are identical for all agents. The utility of an agent k consuming x_1, x_2 is taken to be $u(x_1, x_2) - k$ if the agent invests and $u(x_1, x_2)$ otherwise, where u is homothetic, strictly quasi-concave and differentiable.

2.5 Remarks About the Interpretation of Human Capital in the Model

In our model, human capital investments are *imperfectly observable.* Hence we should not interpret qualified and unqualified workers as being workers with educations of different length. A more consistent way to interpret human capital in our model is to think of workers with the same level of schooling and interpret human capital as what was learned, which depends on effort.

A quantitative exercise using our model would probably need to be extended to allow for both observable and unobservable components of the human capital investments. To some extent one may also think of observable components of human capital being part of the signal θ, but while a signaling rationale for formal education is easy to introduce in our framework, details matter a great deal and one would probably want to allow education to have a direct impact on productivity as well. The introduction of observable components of human capital is a more ambitious project than one may first think and we plan to deal with this in future research.

3 Autarky Equilibrium

For ease of exposition we first describe equilibria in the autarky version of the model. Equilibrium is defined in direct analogy with competitive equilibrium in a perfect information environment. However, the informational problem makes it necessary to use somewhat non-standard notions of wages and labor demands and for clarity we provide a rather detailed definition of equilibrium.

3.1 The Problem of Consumer/Workers

When wages are realized the only thing left to do for a consumer/worker is to allocate her earnings between the two goods. We assume that the utility function over consumption goods is strictly quasi-concave, so the problem

$$\begin{aligned} &\max_{x_1,x_2} u\left(x_1,x_2\right) \\ &\text{subject to } p_1x_1+p_2x_2 \le w \end{aligned} \tag{1}$$

has a unique optimal solution. With the usual abuse of notation we denote by $x_1(w,p), x_2(w,p)$ demand functions, which are identical for all agents in the economy. For notational convenience we also let $v(w,p)$ be the maximized utility in (1),

$$v(w,p) = u(x_1(w,p), x_2(w,p)) \tag{2}$$

3.2 The Problem for the Firms

Without loss of generality we assume that there is one firm in each sector that acts competitively. The representative firm in sector i observes the signal θ for each worker, but not the investment decision, and has to decide "how many" workers of each θ to employ in each task. Formally the firm chooses a pair $\langle l_i^c, l_i^s \rangle$, where $l_i^t : \Theta \to R_+$ is restricted to be integrable for $t = c, s$[5] and the associated inputs of labor in the two tasks are

$$\begin{aligned} s_i &= \int l_i^s(\theta)\, d\theta \\ c_i &= \int l_i^c(\theta)\, P(\theta,\pi)\, d\theta, \end{aligned} \tag{3}$$

where

$$P(\theta,\pi) \equiv \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1-\pi) f_u(\theta)}, \tag{4}$$

Hence, the input of labor in the simple task, s_i, is simply the mass of workers in that job. For the complex task, only those who can perform the task successfully are counted and since for each θ, the posterior probability that the worker is productive $P(\theta,\pi)$ is also the expected fraction of productive workers among those who have signal θ we obtain the expression for c_i in (3) by assuming that a law of large number applies.

Since $P(\theta,\pi)$ is strictly increasing in θ any competitive analysis requires that wages also depend on θ: unless workers with different expected productivity are paid different wages it is impossible for the markets to clear. We therefore let wages be a function $w : \Theta \to R_+$ and assume that the representative firm in sector $i = 1, 2$ take this *wage schedule* as well as the output price p_i as given and solves the profit maximization problem,

$$\max_{\{l_C^i(\cdot), l_S^i(\cdot)\}} p_i y^i \left(\int l_i^c(\theta)\, P(\theta,\pi)\, d\theta, \int l_i^s(\theta)\, d\theta \right) - \int w(\theta) \sum_{t=c,s} l_i^t(\theta)\, d\theta. \tag{5}$$

[5] More generally we could let the firm choose a measure over Θ and our formulation rules out for example any measure with a mass-point. The more general formulation does however not add anything to the analysis.

3.3 Equilibrium Human Capital Investments

Workers/consumers have rational expectations about the wage scheme w and good prices, but faces uncertainty about the realization of the noisy signal when making the investment. The expected utility for an agent with investment cost k is $\int_\theta v(w(\theta), p) dF_q(\theta) - k$ if agent k invests and $\int_\theta v(w(\theta), p) dF_u(\theta)$ otherwise. Rational investment behavior is thus to invest if and only if $\int_\theta v(w(\theta), p) dF_q(\theta) \geq \int_\theta v(w(\theta), p) dF_u(\theta)$ and the corresponding fraction of investors is

$$\pi = G\left(\int_\theta v(w(\theta), p) dF_q(\theta) - \int_\theta v(w(\theta), p) dF_u(\theta)\right) \tag{6}$$

3.4 Conditions for Equilibrium

If all workers behave rationally and a fraction π invests, then it follows from (6) that all workers with costs less than or equal to $G^{-1}(\pi)$ invest and all workers with higher costs do not invest. To avoid excessive notation we therefore leave out the trivial individual investment rules in our definition of equilibrium.

Definition 1 *Output prices $p^* = (p_1^*, p_2^*)$, wages $w^* : \Theta \to R_+$ together the fraction of investors π^* demand functions $x_1(w,p), x_2(w,p)$, outputs (x_1^*, x_2^*) and factor input distributions represented by $\{l_i^{c*}, l_i^{s*}\}_{i=1,2}$ constitutes a competitive equilibrium under autarky if:*

1. *l_i^{c*}, l_i^{s*} solves (5) taking p^* and π^* as given and $x_i^* = y^i\left(\int l_i^{c*}(\theta) P(\theta, \pi^*) d\theta, \int l_i^{s*}(\theta) d\theta\right)$ is the associated output for sector $i = 1, 2$.*

2. *$(x_1(w,p), x_2(w,p))$ solves (1)*

3. *$x_i^* = \int_\theta x_i(w^*(\theta), p^*)(\pi^* f_q(\theta) + (1-\pi^*) f_u(\theta)) d\theta$ for $i = 1, 2$*

4. *$\sum_{i=1,2}(l_i^{c*}(\theta), l_i^{s*}(\theta)) = \pi^* f_q(\theta) + (1-\pi^*) f_u(\theta)$ for all $\theta \in [0,1]$*

5. *$\pi^* = G\left(\int_\theta v(w^*(\theta), p^*) dF_q(\theta) - \int_\theta v(w^*(\theta), p^*) dF_u(\theta)\right)$, where $v(w,p)$ is defined in (2).*

The first condition says that factor demands and outputs must be optimal for the firm given output and factor prices and individual investment behavior, the second that each individual agent must choose her utility maximizing consumption bundle given the income received in equilibrium and prices, the third condition states that the goods market clears given these profit and utility maximizing supply and demand decisions and the fourth condition says that the factor market must clear. Finally, the last condition says that human capital investments must be individually optimal given the equilibrium wage scheme and relative prices.

3.5 Some Technical Lemmas

Before characterizing equilibria it is useful to take a detour and ask what effective factor combinations and outputs could be achieved by a centralized planning agency. For sectors $i = 1, 2$ we let c_i and s_i denote generic inputs of labor in the complex and simple task respectively. We first observe that for the factor input vector $z = (c_1, s_1, c_2, s_2)$ to be feasible when a fraction π of the workers invest in human capital there must exist some "labor demand" $l : \Theta \to R_+^4$ such that

$$\begin{aligned} c_i &= \int_\theta l_i^c(\theta) P(\theta, \pi) \, d\theta \quad \text{for } i = 1, 2 \\ s_i &= \int_\theta l_i^s(\theta) \, d\theta \quad \text{for } i = 1, 2 \\ \sum_i l_i^c(\theta) + \sum_i l_i^s(\theta) &\leq \pi f_q(\theta) + (1 - \pi) f_u(\theta) \end{aligned} \tag{7}$$

The set of feasible factor inputs, which we denote by $Z(\pi)$, is thus

$$Z(\pi) = \left\{ c_1, s_1, c_2, s_2 \middle| \, \exists l : \Theta \to R_+^4 \text{ such that (7) holds} \right\}. \tag{8}$$

From this set of feasible factor inputs we can define the production possibilities set for any given investment behavior in the obvious way as

$$X(\pi) = \left\{ (x_1, x_2) \in R_+^2 \, \middle| x_i = y^i(c_i, s_i) \text{ for some } (c_1, s_1, c_2, s_2) \in Z(\pi) \right\}. \tag{9}$$

Convexity properties of $X(\pi)$ and $Z(\pi)$ are important for the analysis and for future reference we list the relevant results that are used in later sections.

Lemma 1 *The set of feasible factor inputs in the economy is given by*

$$Z(\pi) = \left\{ (c_1, s_1, c_2, s_2) \in R_+^4 \, | g(c_1 + c_2, s_1 + s_2; \pi) \geq 0 \right\}, \tag{10}$$

where

$$g(c, s; \pi) \equiv \pi - c - s + (1 - \pi) F_u \left(F_q^{-1} \left(\frac{\pi - c}{\pi} \right) \right), \tag{11}$$

Lemma 2 *$g(c, s; \pi)$ is strictly quasi-concave in (c, s) for any given $\pi > 0$*

Lemma 3 *$Z(\pi)$ is convex for every $\pi \in [0,1]$.*

Lemma 4 *Suppose that y^i is concave for $i = 1,2$. Then, $X(\pi)$ is a convex set for every $\pi \in [0,1]$.*

Lemma 5 *If in addition to the hypotheses in Lemma 4 the factor intensity assumption A1 is satisfied, then for each $x', x'' \in X(\pi)$ where $x', x'' >> 0$ and each $\lambda \in (0,1)$ there is a neighborhood B of $\lambda x' + (1-\lambda) x''$ such that $x \in X(\pi)$ for all $x \in B$ (that is, the frontier of $X(\pi)$ can be described by a strictly concave downward sloping function as in Figure 1).*

The crucial feature of the model that drives all these convexity properties is that when workers are moved from the complex to the simple task, the larger is the input of labor in the complex task, the lower is the probability that the marginal worker is qualified. This results in a convex set of feasible factor inputs. Combined with a technology that is convex in the factor inputs, this gives convexity in $X(\pi)$ as well.

3.6 Equilibrium Characterization

We begin the analysis by treating π as a fixed parameter. For lack of better language we will call allocations and prices that satisfy all equilibrium conditions except for condition 5 in Definition 3.4 "continuation equilibria". Given any π, there is a unique continuation equilibrium, which is characterized in a way that allows us to reduce the final equilibrium condition, that π must be consistent with rational investment behavior, to a simple fixed point equation.

All agents have identical homothetic utility functions over consumption bundles, so the economy is like a representative agent economy when investment costs are sunk. While the distribution across agents is indeterminate we can characterize which combinations of aggregate consumption of each good that are consistent with (restricted) Pareto optimality by solving

$$\begin{aligned} &\max_{x_1,x_2} u(x_1,x_2) \\ &\text{subject to.}\,(x_1,x_2) \in X(\pi) \end{aligned} \tag{12}$$

$X(\pi)$ is convex and compact and u is strictly quasi-concave, so (12) has a unique solution fully characterized as a tangency between the production possibilities set and the highest achievable level curve to u as depicted in Figure 1.

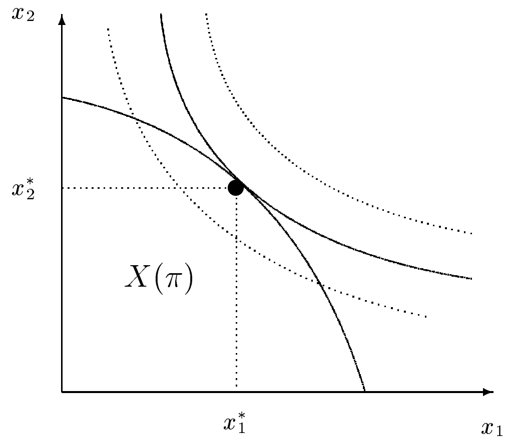

Figure 1: Efficient Production and Consumption Given Investment Behavior

There is a distortion in the model, captured in the "restricted" production possibilities set $X(\pi)$ that is strictly contained within the full information production possibilities set due to the asymmetric information. Nevertheless, given our competitive assumptions and all the convexity in the model there seem to be no reasons for why the competitive model should not produce *informationally constrained Pareto Optima.* That is, one would think that equilibrium outputs would coincide with the solution to (12) and that relative prices would be given by the tangency condition in Figure 1. Our first proposition confirms that this guess is right and also provides a natural characterization of the competitive wages. Define $\theta(c,\pi)$ as the threshold signal needed in order to generate a labor input c in the complex task when a fraction π invests, that is

$$\theta(c,\pi) \equiv F_q^{-1}\left(\frac{\pi - c}{\pi}\right). \tag{13}$$

Proposition 1 *Aggregate outputs (x_1^*, x_2^*) and prices (p_1^*, p_2^*) are consistent with equilibrium conditions 1-4 of the model if and only if (x_1^*, x_2^*) solves (12) and (p_1^*, p_2^*) is a normal to a hyperplane that separates $X(\pi)$ and the set of bundles such that $u(x_1, x_2) \geq u(x_1^*, x_2^*)$. Moreover, the equilibrium wages must satisfy*

$$w^*(\theta) = \begin{cases} p_i^* \frac{\partial y^i(c_i^*, s_i^*)}{\partial s_i} & \theta \leq \theta(c_1^* + c_2^*, \pi) \\ p_i^* P(\theta,\pi) \frac{\partial y^i(c_i^*, s_i^*)}{\partial c_i} & \theta > \theta(c_1^* + c_2^*, \pi) \end{cases}, \tag{14}$$

where $(c_1^*, c_2^*, s_1^*, s_2^*)$ are effective factor inputs consistent with outputs (x_1^*, x_2^*) and the threshold signal $\theta(c_1^* + c_2^*, \pi)$ must satisfy

$$P(\theta(c_1^* + c_2^*, \pi), \pi) \frac{\partial y^i(c_i^*, s_i^*)}{\partial c_i} = \frac{\partial y^i(c_i^*, s_i^*)}{\partial s_i} \tag{15}$$

A rigorous proof is in the appendix, but since we use Proposition 1 extensively in the rest of the paper we provide a rather detailed heuristic argument. Interpret $P(\theta,\pi)$ as "efficiency units" of labor provided by a worker with signal θ if employed in the complex task. Clearly, there would be arbitrage possibilities unless $w(\theta) = w_c P(\theta,\pi)$ for all θ employed in the complex task and $w(\theta) = w_s$ for all θ employed in the simple task. Moreover, $w(\theta) = w_c P(\theta,\pi) \geq w_s$ for all θ employed in the complex task, since otherwise the workers in the simple task could be replaced by cheaper workers. Similarly, $w_s \geq w_c P(\theta,\pi)$ for θ in the simple task since otherwise the workers in the complex task could be replaced by cheaper workers. Now, $P(\theta,\pi)$ is monotonically increasing in θ, so we conclude that in equilibrium there is a threshold θ^*, which satisfies $w_s = w_c P(\theta^*,\pi)$, such that workers above (below) the threshold are assigned to the complex (simple) task. Given these arbitrage conditions on wages, the problem for the representative firm in each sector reduces to $\max_{c_i,s_i} p_i y^i(c_i,s_i) - w_c c_i - w_s s_i$, so equilibrium requires that

$$p_i \frac{\partial y^i(c_i^*, s_i^*)}{\partial c_i} = w_c \text{ and } p_i \frac{\partial y^i(c_i^*, s_i^*)}{\partial s_i} = w_s, \tag{16}$$

and combining (16) with $w_s = w_c P(\theta^*,\pi)$ this gives condition (15).

Next, observe that since the utility function is homothetic aggregate consumption must be the solution to

$$\begin{aligned} &\max_{x_1,x_2} u(x_1,x_2) \\ &\text{subject .to } p_1^* x_1 + p_2^* x_2 \leq \int w^*(\theta) f_\pi(\theta)\, d\theta. \end{aligned} \tag{17}$$

The budget constraint binds and zero profits implies that the right hand side of the constraint in (17) equals $p_1^* x_1^* + p_2^* x_2^*$. Combining with (16) it follows that if conditions on u are imposed that guarantees that (17) has an interior solution, then

$$\frac{\frac{\partial u(x_1^*,x_2^*)}{\partial x_1}}{\frac{\partial u(x_1^*,x_2^*)}{\partial x_2}} = \frac{p_1^*}{p_2^*} = \frac{\frac{\partial y(c_1^*,s_1^*)}{\partial c_1}}{\frac{\partial y(c_2^*,s_2^*)}{\partial c_2}} = \frac{\frac{\partial y(c_1^*,s_1^*)}{\partial s_1}}{\frac{\partial y(c_2^*,s_2^*)}{\partial s_2}} = \frac{dx_1(x_2)}{dx_2}. \tag{18}$$

Thus, in equilibrium the relative prices must separate $X(\pi)$ and the set of better bundles for the fictitious representative consumer.

Proposition 1 immediately implies that equilibria are unique in all relevant respects.

Corollary 1 *Given any $\pi > 0$ there is a unique aggregate bundle (x_1^*, x_2^*) that is consistent with equilibrium and equilibrium prices and wages are unique up to a multiplicative constant.*

Uniqueness of the aggregate bundle follows directly from Proposition 1 since (12) has a unique solution. This determines a unique relative price between the goods. In the appendix we prove that factor ratios are also unique, implying that equilibrium prices are unique up to the choice of units and that equilibria are fully characterized in terms of the solution to (12).

3.7 Equilibrium Investments

We choose good 2 as our unit of account and let $p(\pi)$ be the equilibrium price of good one. Furthermore, we let $x_1(\pi), x_2(\pi)$ be the equilibrium outputs, $c_i(\pi), s_i(\pi)$ the (unique) equilibrium factor inputs, $\widetilde{\theta}(\pi) \equiv F_q^{-1}\left(\frac{\pi - c_1(\pi) - c_2(\pi)}{\pi}\right)$ the associated (unique) threshold signal and $r_i(\pi) = c_i(\pi)/s_i(\pi)$ the corresponding factor ratio given a fraction of investors π. We may then write the unique equilibrium wage scheme $w(\theta;\pi)$ as

$$w(\theta;\pi) = \begin{cases} p(\pi)\frac{\partial y^1(r_1(\pi),1)}{\partial s} = \frac{\partial y^2(r_2(\pi),1)}{\partial s} & \text{for } \theta \leq \widetilde{\theta}(\pi) \\ p(\pi)P(\theta,\pi)\frac{\partial y^1(r_1(\pi),1)}{\partial c} = P(\theta,\pi)\frac{\partial y^2(r_2(\pi),1)}{\partial c} & \text{or } \theta > \widetilde{\theta}(\pi) \end{cases}. \tag{19}$$

If the final equilibrium condition (6) is satisfied for $w(\theta;\pi)$ and $p(\pi)$ generated above then all equilibrium conditions are satisfied, while if this is not the case, then the economy can not be in equilibrium for that particular fraction of investors. The equilibria of the model are thus fully characterized as fixed points to

$$\pi = G\left(\int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_q(\theta) - \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_u(\theta)\right), \tag{20}$$

where $v(w,p)$ is defined in (2). For ease of notation we define the equilibrium benefits of investment,

$$B(\pi) \equiv \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_q(\theta) - \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_u(\theta). \tag{21}$$

We summarize the important properties of the function B as a proposition:

Proposition 2 *The function B defined in (21) satisfies the following properties: 1) B is continuous in π, 2) $B(0) = 0$, 3) $B(1) = 0$, and 4) $B(\pi) > 0$ for all $\pi \in (0,1)$.*

This result should be highly intuitive. For intermediate values of π investors are better off than workers who don't invest since investing increases the probability of a high signal and thereby the probability of a higher wage. The only expectations are when π is 0 or 1, in which case the posterior probability of investment and therefore also the wage is constant in the signal. Using Proposition

2, existence of equilibrium follows trivially from the intermediate value theorem and if $G(0)>0$ any equilibrium much be non-trivial.

4 Trade

We now assume that two countries, h, f trade in goods on a frictionless market, but that workers are unable to cross national borders. We let λ_h and $\lambda_f = 1-\lambda_h$ denote the fractions of workers in each country. We write $w_j : \Theta \rightarrow R_+$ for the wages in country j and π_j for the fraction of investors and abuse previous notation by letting $\pi = (\pi_h, \pi_f)$ be the vector of fractions of investors rather than a scalar. Outputs are denoted x_{ij} where the first index refers to the good ($i=1,2$) and the second to the country and $x = (x_{1h}, x_{2h}, x_{1f}, x_{2f})$ denotes the vector of outputs. When factor input distributions are needed explicitly we add a country index and write l_{ij}^t for the labor demand in sector i, country j and task t.

4.1 Trade Equilibrium

Equilibrium is defined as in Definition 1, except that all variables except for goods prices now are indexed by country. The only condition that needs any modification is the goods market clearing conditions, which with international markets become

$$\sum_{j=h,f} \lambda_j x_{ij}^* = \sum_{j=h,f} \int_\theta x_i\left(w_j^*(\theta), p^*\right) \lambda_j f_{\pi^j}(\theta)\, d\theta \tag{22}$$

for each good i.

The production possibilities set in a country with λ_j workers and fraction of investors π^j is simply $\lambda_j X\left(\pi^j\right)$, where X is defined as in (9). The world production possibilities are thus by

$$X_w(\pi) = \lambda_h X(\pi_h) + \lambda_f X(\pi_f), \tag{23}$$

which inherits all relevant properties from the production possibilities set of the autarky model. In particular, since a linear combination of convex sets is convex we have that;

Lemma 6 *$X_w(\pi)$ is convex. Moreover, if the factor intensity assumption **A1** is satisfied, then for each $x', x'' \in X_w(\pi)$ where $x', x'' >> 0$ and each $\lambda \in (0,1)$ there is a neighborhood B of $\lambda x' + (1-\lambda) x''$ such that $x \in X_w(\pi)$ for all $x \in B$*

The characterization of equilibrium also follows the model without trade closely and the analogue to Proposition 1 is;

Proposition 3 *For any given investments* π, *world outputs* x_w^* *and prices* p^* *and are consistent with equilibrium if and only if* x_w^* *solves (12) and* p^* *is a normal to a hyperplane that separates* $X_w(\pi)$ *and the set of bundles such that* $u(x_1, x_2) > u(x_1^{w*}, x_2^{w*})$. *Moreover, the equilibrium wages must satisfy*

$$w_j^*(\theta) = \begin{cases} p_i^* \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial s_i} & \text{for } \theta \leq \theta\left(c_{1j}^* + c_{2j}^*, \pi_j\right) \\ p_i^* P(\theta, \pi) \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial c_i} & \text{or } \theta > \theta\left(c_{1j}^* + c_{2j}^*, \pi_j\right) \end{cases}, \tag{24}$$

for each good i *that is produced in country* j, *where* $(c_{1j}^*, c_{2j}^*, s_{1j}^*, s_{2j}^*)$ *are effective factor inputs consistent with outputs* x_j^* *and where* $x_w^* = \lambda_h x_h^* + \lambda_f x_f^*$. *Finally, the threshold signals* $\theta\left(c_{1j}^* + c_{2j}^*, \pi_j\right)$ *must satisfy*

$$P\left(\theta\left(c_{1j}^* + c_{2j}^*, \pi_j\right), \pi_j\right) \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial c_i} = \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial s_i} \tag{25}$$

Except for the trivial indeterminacy that arises in the model since there are many different labor demands that give the same effective factor inputs the equilibrium is unique.

Proposition 4 *Given any* π *the world output* x_w^* , *the country specific outputs* x_j^* *and factor inputs* $(c_{1j}^*, c_{2j}^*, s_{1j}^*, s_{2j}^*)$ *are all uniquely determined. Output prices and wages are also unique up to a multiplicative constant.*

4.2 Factor Prices and Trade Flows

Given a fixed fraction of investors in each country, the model reduces to something very similar to the standard $2 \times 2 \times 2$ trade model. The only real difference is that the empirical content in our predictions are different. In particular, we can directly import the logic behind factor price equalization to our model, but that result only says that *effective factor prices* are the same. Also, our model predicts that the wages in the export sector in a rich (poor) country should be higher (lower) than the average wage in the country

Define the "effective factor prices" as

$$w_{sj}^* = p_i^* \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial s_i} \text{ and } w_{cj}^* = p_i^* \frac{\partial y^i\left(c_{ij}^*, s_{ij}^*\right)}{\partial c_i}. \tag{26}$$

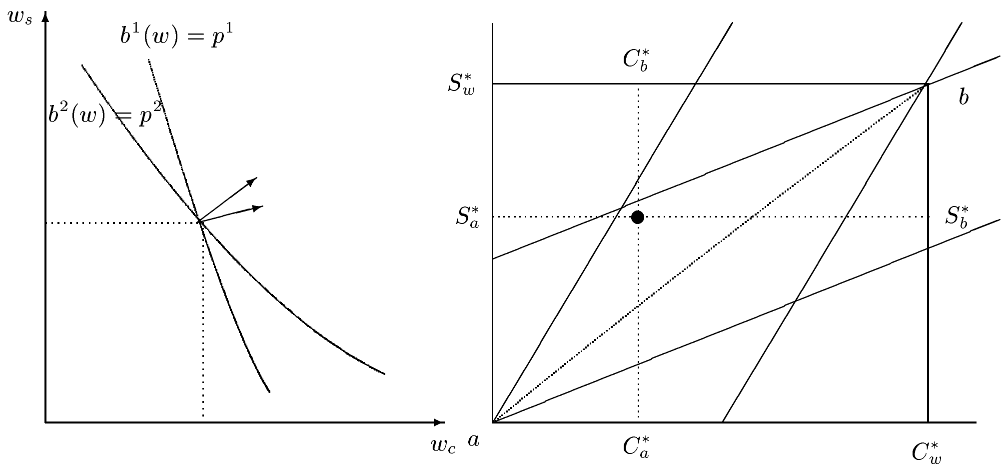

Figure 2: "Quasi" Factor Price Equalization

Applying (24) we see that the arbitrage conditions on wages imply that the profit maximization problem for a firm in country j and sector i reduces to $\max_{c_i,s_i} p_i^* y^i(c_{ij}, s_{ij}) - w_{cj}^* c_i - w_{sj}^* s_i$, identical to the firm problem in the traditional $2 \times 2 \times 2$ model. Results from that model that don't rely on factor endowments of factors can therefore be directly imported, in particular:

Proposition 5 (Quasi-factor price equalization) *If both countries produce both goods, then (effective) factor prices are equalized, i.e., $(w_{ch}^*, w_{sh}^*) = (w_{cf}^*, w_{sf}^*)$. If country h does not produce good 1 , then $\frac{w_{ch}^*}{w_{sh}^*} \geq \frac{w_{cf}^*}{w_{sf}^*}$, while if country h does not produce good 2 the inequality is reversed.*

It is instructive to consider Figure 2 to see to what extent our model corresponds to the standard theory and to what extent we are doing something different. The graphs are drawn for the case when effective factor prices are equalized. In the graph to the left, $b^1(w)$ and $b^2(w)$ are the minimized costs to produce one unit of output in each industry and the level curves depicted are thus depicting combinations of w_c and w_s such that there are zero profits in both industries (taking goods prices as given). We can show that the single-crossing condition on the isoquants to $y^1(c,s)$ and $y^2(c,s)$, assumption **A1** on page 6, implies that the "zero profit contour" is always steeper for industry one. The idea is simply that since complex labor is more important in the production of the high tech good a larger decrease in the wage for simple labor is needed to compensate for a given increase in the wage for complex labor. The basic insight from the graph is that the fact that the curves crosses only once means that there for each configuration of output prices there is a unique set of (effective) factor prices that is consistent with both sectors producing positive output. It follows immediately that (effective) factor prices must be equalized if both sectors are active in both countries[6].

The point with the graph to the right is that in spite of the fact that factors are endogenous we can apply the usual criterion for when factor prices will be equalized. The basic insight is that when an "integrated equilibrium" for the world is derived, effective world factor inputs (C_w^* and S_w^* in figure), world uses of each factor in each industry, and *effective factor contributions from each country* (C_j^* and S_j^* for $j = a, b$ in figure) are simultaneously determined in this equilibrium. This last point follows because each point of the "world factor frontier" corresponds with unique factor inputs from each country since each point on the world factor frontier requires that the marginal worker in each country is equally likely to be qualified. Hence, the usual graph that depicts the region for factor price equalization as the area within parallelogram the can be drawn (in the figure, the effective factor use in the high tech sector is in the graph as the flatter line from the southwest origin to the intersection with the steeper line, or as the flatter line from the northeast origin to the intersection with the other steep line, and the factor use in the low tech sector is the vector from the intersection to the opposite origin.)

Observe that average wage payments are *not* equalized across countries unless investment behavior is the same in both countries due to differences in human capital distributions. In our simple model, this results in lower wages in the more demanding jobs since workers are productive with a lower probability.

The idea that labor in different countries have different productivities has been explored previously in international trade, notably by Trefler [20], [21]. Our approach is however quite different. We have a more explicit structure that determines human capital differences, so our model generates additional implications for cross country comparisons of wage distributions that we intend to explore further in a sequel to this paper. For example, the average wage in the export sector of a rich country should be higher than the average wage in the domestic sector. We also get the seemingly counterfactual prediction that there should be more variability in wages in a rich than in a poor country[7].

[6] Since the analysis here is reduced to a well-known problem in international trade we have omitted several details. See Dixit and Norman [3] for a more careful treatment.

[7] Observe here that our model lacks observable human capital. Naively, this prediction is about "unexplained" variation in wages. This is however not so straightforward: details matter a great deal when observable human capital

The "factor content" of trade goes in the expected direction, as should be clear from Figure 2:

Proposition 6 (factor abundance hypothesis) *If $\pi_h < \pi_f$, then country h is a net importer of the skill intensive good and country f is a net exporter of the skill intensive good.*

While the result seems obvious, Proposition 6 is not a direct application of well-known results in the same way as Proposition 5 and some work is needed in the proof.

4.3 How Effective Factor Prices Respond when Investments Change

Intuitively we expect an increase in the fraction of investors in the other country to increase the production of high tech goods in that country and decrease the production of high tech goods at home, as well as make the high tech good relatively cheaper. Hence, it seems that the need for complex labor is reduced in the country where the fraction of investors is unchanged, which should reduce the relative price for complex labor and therefore reduce incentives.

As before, good 2 is the numeraire and $p(\pi)$ is the equilibrium price of good 1. We let $x_{1j}(\pi), x_{2j}(\pi)$ be the equilibrium outputs, $c_{ij}(\pi), s_{ij}(\pi)$ the (unique) equilibrium factor inputs, $\theta_j(\pi) \equiv F_q^{-1}\left(\frac{\pi - c_{1j}(\pi) - c_{2j}(\pi)}{\pi}\right)$ the associated (unique) threshold signal and $r_{ij}(\pi) = c_{ij}(\pi)/s_{ij}(\pi)$ the corresponding factor ratios. We may then write the unique equilibrium wage scheme in country j as

$$w_j(\theta;\pi) = \begin{cases} p(\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial s_1} = \frac{\partial y^2(r_{2j}(\pi),1)}{\partial s_2} & \text{for } \theta \leq \theta_j(\pi) \\ p(\pi)P(\theta,\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial c_1} = P(\theta,\pi)\frac{\partial y^2(r_{2j}(\pi),1)}{\partial c_2} & \text{or } \theta > \theta_j(\pi) \end{cases}. \quad (27)$$

For any π and $j = h, f$ we let $c_j(\pi) = c_{1j}(\pi) + c_{2j}(\pi)$ and $s_j(\pi) = s_{1j}(\pi) + s_{2j}(\pi)$ and let

$$\begin{aligned} w_{jc}(\pi) &= p(\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial c_1} = \frac{\partial y^2(r_{2j}(\pi),1)}{\partial c_2} \\ w_{js}(\pi) &= p(\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial s_1} = \frac{\partial y^2(r_{2j}(\pi),1)}{\partial s_2} \end{aligned} \quad (28)$$

be the effective equilibrium factor prices (expressed in units of good 2). We begin with making a simple "revealed profit maximization" argument that gives us some discipline on how the correlation of factor prices and effective factor uses relates to the correlation between prices and outputs.

is introduced. One reason is that a natural signaling motive for observable human capital is created when there are both perfectly observable and imperfectly observable components of human capital.

Lemma 7 *For any $\pi \neq \pi'$ we have that*

$$(p(\pi) - p(\pi'))(x_{1j}(\pi) - x_{1j}(\pi')) \geq (w_{jc}(\pi) - w_{jc}(\pi'))(c_j(\pi) - c_j(\pi')) \qquad (29)$$
$$+ (w_{js}(\pi) - w_{js}(\pi'))(s_j(\pi) - s_j(\pi'))$$

Moreover, if $r_{ij}(\pi) \neq r_{ij}(\pi')$ for some industry i, the inequality is strict.

The main usefulness of the result is that combined with optimal consumer behavior and the characterization of equilibrium wages it guarantees the expected comparative statics results for how the effective factor prices depend on how many agents invest in the economy.

Proposition 7 *Suppose that π, π' are such that $x_{ij}(\pi), x_{ij}(\pi') > 0$ for all i, j (which implies that effective factor prices are equalized across countries) and suppose that $\pi_h < \pi'_h$ and $\pi_f \leq \pi'_f$. Then $w_c(\pi) > w_c(\pi')$ and $w_s(\pi) < w_s(\pi')$.*

4.4 Cross Country Effects on Incentives

In analogue with the autarky model, the benefits of investment in country j are

$$B^j(\pi) \equiv \int_\theta v(w_j(\theta;\pi), p(\pi))\, dF_q(\theta) - \int_\theta v(w_j(\theta;\pi), p(\pi))\, dF_u(\theta). \qquad (30)$$

For simplicity we assume that preferences are such that agents are risk neutral in money income. We have already assumed that u is homothetic and from standard analysis of risk preferences it then follows that u must be homogenous of degree one, implying that the value function corresponding to the optimal consumption plan satisfies

$$v(w,p) = u(x_1(w,p), x_2(w,p)) = wu(x_1(1,p), x_2(1,p)) = v(1,p) \qquad (31)$$

Hence we may write the benefits of investment as

$$B^j(\pi) = v(1, p(\pi))(w_{js}(\pi)\left(F_q\left(\widetilde{\theta}(\pi)\right) - F_u\left(\widetilde{\theta}(\pi)\right)\right) + w_{jc}(\pi)\int_{\widetilde{\theta}(\pi)}^1 P(\theta,\pi)(f_q(\theta) - f_u(\theta))\, d\theta). \qquad (32)$$

Using Proposition 7 and the equilibrium condition (25) it is easy to check that the bracketed expression is decreasing in $\pi^k \neq \pi^j$ by straightforward differentiation. However, it is equally clear that $v(1, p(\pi))$ is increasing in π^k, so (32) suggests that the net effect is ambiguous.

In our parametric examples in Section 5 below the cross effect on incentives is always negative. This is also the case in our related work on economics of discrimination in Moro and Norman [10]. It is conceivable that this is true in general and that it can be shown by exploiting the relationships between goods and factor prices, but we have not managed to do so yet. However, already by looking at (32) we can see that incentives are affected asymmetrically in the two countries when investments change in one country.

5 A Simple Parametrization of the Model

To illustrate how our model is capable of generating specialization and inequalities in the most transparent way we now consider a simplified version of the model. We assume that skilled labor is the only input for production of the high tech good and that unskilled labor is the only input for production of the low tech good. The set of signals is $\Theta = \{b, g\}$ and the distributions conditional on the investment decision are symmetric. Finally, preferences are Cobb Douglas and costs of investment are uniformly distributed. In sum:

<table>
<tr><td>Production technology</td><td>$y^1(c) = c$
$y^2(s) = s$</td></tr>
<tr><td>Conditional Signal Probabilities</td><td><table><tr><td></td><td>b</td><td>g</td></tr><tr><td>if invest</td><td>$1-\eta$</td><td>η</td></tr><tr><td>if don't invest</td><td>η</td><td>$1-\eta$</td></tr></table> $\eta > 1/2$</td></tr>
<tr><td>Utility function (ignoring investment)</td><td>$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$</td></tr>
<tr><td>Distribution of investment costs</td><td>$G(k) = \frac{1}{\overline{k}-\underline{k}}, k \in [\underline{k}, \overline{k}]$</td></tr>
</table>

(33)

5.1 Autarky Equilibrium

As in the general model we use good 2 as the numeraire. Given the Cobb Douglas preferences in (33) the individual demands as a function of the wage and the price of good 1 are

$$\begin{aligned} x_1(p_1, 1, w) &= \frac{\alpha w}{p_1} \\ x_2(p_1, 1, w) &= (1-\alpha) w, \end{aligned} \tag{34}$$

and the corresponding maximized utility is $v(p_1, 1, w) = w\alpha^{\alpha}(1-\alpha)^{1-\alpha}/p_1^{\alpha}$.

Workers rationally predict p_1 and the equilibrium in the investment stage. Wages with only two signals may be described as a pair (w_b, w_g), where w_b (w_g) is the wage earned by a worker that draws a bad (good) signal. Workers who invest get signal g with probability η and b with probability $1-\eta$, while workers who do not invest get signal g with probability $1-\eta$ and b with probability η. Computing the expectation of $v(w,p)$ *conditional on investment* and subtracting from this the expectation of $v(w,p)$ *conditional on not investing* we get what we refer to as the *gross benefits of investment,*

$$E\left\{v\left(w,p\right)|\text{inv}\right\} - E\left\{v\left(w,p\right)|\text{ no inv}\right\} = \frac{(2\eta-1)(w_g-w_b)}{\left(p_1\right)^{\alpha}}\alpha^{\alpha}(1-\alpha)^{1-\alpha}. \tag{35}$$

Here, w_g is the wage earned by a worker with the good signal and w_b is the wage of a worker with a bad signal. Rational investments by workers thus require that

$$\pi = G\left(\frac{(2\eta-1)(w_g-w_b)}{\left(p_1\right)^{\alpha}}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right), \tag{36}$$

which corresponds with condition 5 in the general definition of equilibrium on page 9.

For the same reasons as in the general model there is a unique "continuation equilibrium" which pins down the allocation and the relative prices given any investment behavior. Depending on the fraction of investors and on the parameter of the utility function this unique equilibrium can take on three different forms. All derivations are in Moro and Norman [11].

Equilibria of type A (allocation of workers in "accordance with signals"): all workers with signal b (g) are in the low (high) tech sector. If this is an equilibrium the corresponding outputs are

$$x_1 = c = \eta\pi \quad \text{ and } \quad x_2 = s = (1-\eta)\,\pi + \eta\,(1-\pi) \tag{37}$$

The price of good 1 must be such that consumers are willing to consume the quantities in (37), which by use of (34) implies that

$$p_1 = \frac{\alpha}{1-\alpha}\frac{(1-\eta)\,\pi + \eta\,(1-\pi)}{\eta\pi}. \tag{38}$$

Finally, factor market equilibrium requires that the firms earn zero profits on each worker. Hence $w_b = 1$ and

$$w_g = p_1\frac{\pi\eta}{\pi\eta + (1-\eta)\,(1-\pi)}. \tag{39}$$

The expression (39) should be intuitive: $\pi\eta/(\pi\eta + (1-\eta)\,(1-\pi))$ is the posterior probability that a worker with signal g produces a unit of the high tech good, so the condition says that the wage equals the expected value of the output from a worker. Besides the conditions above we must also check that: 1) firms in the high tech sector have no incentive to use workers with bad signals and, 2) firms in the low tech sector have no incentive to hire workers with good signals. These conditions hold if and only if $1-\eta \leq \alpha \leq \eta$ and $\pi \leq (\alpha+\eta-1)/(2\eta-1)$.

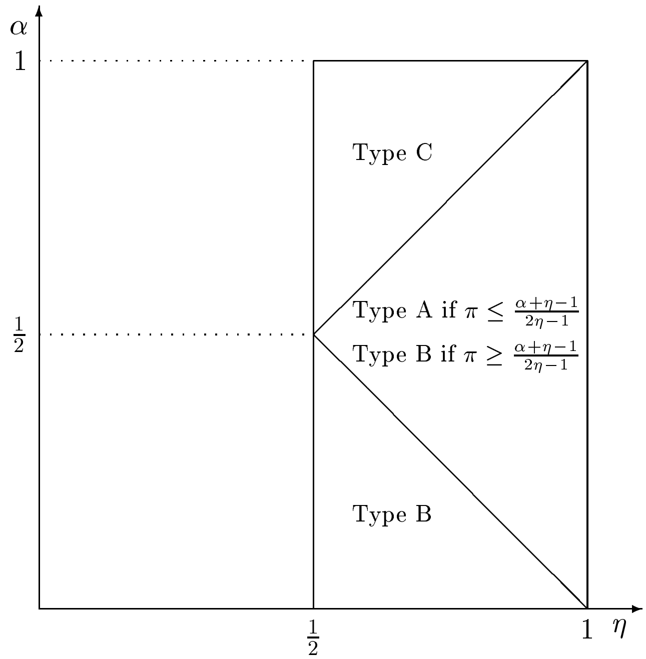

Figure 3: Types of autarky equilibria in the (η, α) space

Equilibria of type B (mixing of good signals): a fraction $\gamma \in (0,1)$ of the workers with signal g are in the high tech sector and the remaining g workers and all workers with signal b are in the low tech sector. Firms must then offer the same wage to workers with signal g independently of sector assignment and, since all workers in the low tech sector must be paid a wage of 1, it follows that $w_g = w_b = 1$ in such an equilibrium. Moreover, there must be zero profits in the high skill sector as well, implying that

$$1 = p_1 \frac{\pi\eta}{\pi\eta + (1-\eta)(1-\pi)}. \tag{40}$$

The equilibrium price p_1 is thus given by (40), so the goods market clearing condition (38) now determines the *outputs* that makes consumers willing to purchase the outputs on the market, given the price that now is determined on the "supply side". This type of equilibrium thus requires that there exists some $\gamma \in (0,1]$ such that

$$\frac{\alpha}{(1-\alpha)p_1} = \frac{x_1}{x_2} = \frac{\gamma\eta\pi}{(1-\gamma)\eta\pi + (1-\eta)\pi + \eta(1-\pi)} \tag{41}$$

for p_1 solving (40). This is the case whenever $1-\eta \leq \alpha \leq \eta$ and $\pi > (\alpha+\eta-1)/(2\eta-1)$ or if $\alpha < 1-\eta$. That is, if π is large or α small, then workers with signal g are in both sectors.

Equilibria of type C (mixing of bad signals): a fraction $\beta \in (0,1)$ of b workers and all g workers are allocated to the high tech sector, and the remaining b workers are allocated to the low tech sector. In this case, workers with signal b must be equally valuable in each sector, implying that

$$p_1 \frac{(1-\pi)\eta}{(1-\pi)\eta + (1-\eta)\pi} = w_b = 1. \tag{42}$$

To understand the condition, observe that $(1-\pi)\eta/((1-\pi)\eta+(1-\eta)\pi)$ is the posterior probability that a worker with signal b is productive in the high tech sector. The price p_1 is determined by (42) and w_g is then obtained by substituting p_1 into (39). The only question that remains is whether there exists $\beta \in (0,1]$ such that the goods market clears, that is solving

$$\frac{\alpha}{(1-\alpha)p_1} = \frac{\eta\pi + \beta(1-\eta)\pi}{(1-\beta)((1-\eta)\pi + \eta(1-\pi))}, \tag{43}$$

where, again, p_1 is the unique solution to (42). This is satisfied whenever $\alpha > \eta$.

Figure 3 shows the regions in the parameter space for each type of equilibrium. These regions are disjoint, and since it is never an equilibrium to mix both signals, the possibilities are exhaustive. Uniqueness of "continuation equilibria" thus follows directly from the computations sketched above (details are in Moro and Norman [11]).

5.2 Equilibrium investments

From the characterization in Section 5.1 we derive a closed form expression for the incentives to invest given any π by substituting the unique equilibrium wages and prices into the expression for the gross benefits to invest in (35). In the case when $\alpha \leq \eta$, the equilibrium is of type A or type B (depending on π). The corresponding gross benefits to invest as a function of the fraction of investors is (for derivations see Moro and Norman [11])

$$B(\pi) = \max\left\{(2\eta-1)\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\eta}\right)^{\alpha}\left(\frac{\alpha-(\pi\eta+(1-\pi)(1-\eta))}{\pi\eta+(1-\pi)(1-\eta)}\right), 0\right\}. \tag{44}$$

If $\alpha > \eta$ on the other hand side, we have that

$$B(\pi) = (2\eta-1)\left(\frac{\pi(1-\eta)}{\pi(1-\eta)+(1-\pi)\eta}\right)^{\alpha}\left(\frac{\pi+\eta-2\pi\eta}{1-(\pi+\eta-2\pi\eta)}\cdot\frac{\eta}{1-\eta}-1\right)\alpha^{\alpha}(1-\alpha)^{1-\alpha}. \tag{45}$$

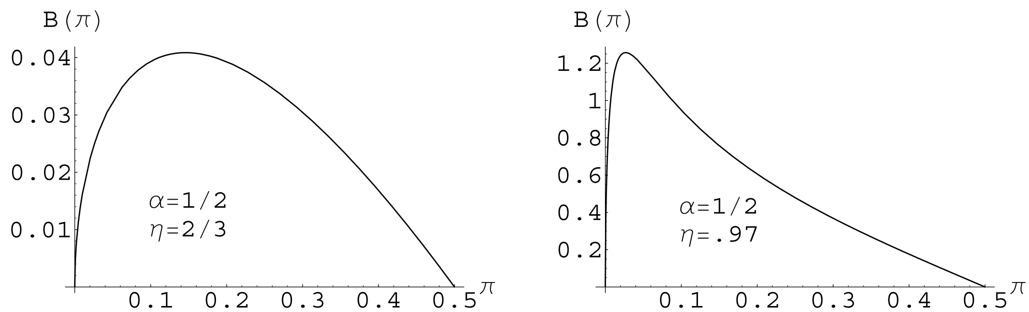

Figure 4: Gross incentives to invest under autarchy

In both cases, any π such that $\pi = G(B(\pi))$ constitutes an equilibrium fraction of investors and the corresponding equilibrium wages, prices and allocation can be backed out from the relevant case of the characterization above. Since $G(B(\pi))$ is continuous and takes on values on $[0,1]$ existence of equilibrium follows trivially.

Figure 4 plots the function $B(\pi)$ for two different sets of parameter values. While $B(\pi)$ is not necessarily concave (see the example to the right), it is always single-peaked and "initially concave".

5.3 Uniqueness of Autarky Equilibria

A useful feature of this parametrization is that there are simple sufficient conditions for when the autarky equilibrium is unique.

Proposition 8 *Given 33, if $G(\cdot)$ is concave, $\alpha \leq \eta$ and $\underline{k} < 0$ then the model displays a unique autarky equilibrium.*

The proof in the appendix exploits that it is impossible for $G(B(\pi))$ to intersect the 45 degree line from below. This explains the condition $\underline{k} < 0$, since then $G(B(0)) > 0$, meaning that the curve is above the 45 degree line initially.

It is instructive to see how the set of equilibria changes as parameters of the model change. Figure 5 plots the full set of equilibria for $\eta = 2/3$, $\alpha = 1/2$ and a distribution of investment costs that is uniform over $[\underline{k}, \overline{k}]$.

In the computations that generate Figure 5 we set $\overline{k} - \underline{k} = 0.2$ and vary $\underline{k}$. For $\underline{k} \leq -0.2$ all agents derive positive utility from investing in human capital, so all agents obviously invest. Then, for $-0.2 < \underline{k} < 0$ we have (as implied by Proposition 8) a unique equilibrium where some

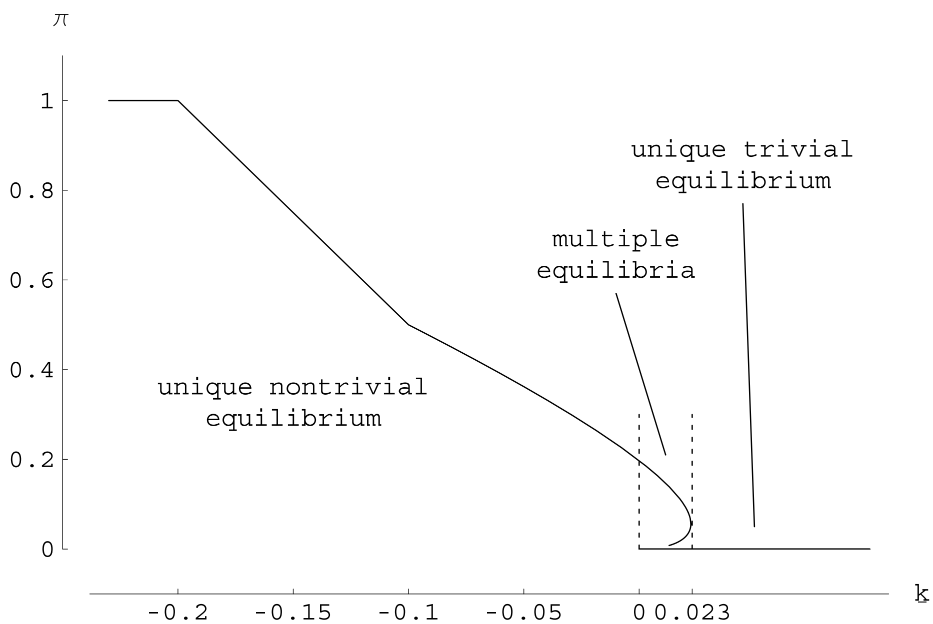

Figure 5: Equilibrium investments in autarky with $\eta = 2/3, \alpha = 1/2$

agents invest and others do not. Naturally, the fraction of investors in the unique equilibrium is decreasing as the cost distribution is shifted towards higher costs. When $\underline{k} \geq 0$ all agents have positive investment costs and for this reason there is now always a trivial equilibrium where nobody invests and there is no production of the high tech goods. However, unless $\underline{k}$ is too big there are also two non-trivial equilibria (the highest of these "stable" in the ad hoc sense). Finally, if $\underline{k}$ is too big there is only the trivial equilibrium.

An alternative way at looking at this is illustrated in Figure 6, which plots the fixed point equation $\pi = G(B(\pi))$ that determines the set of equilibria for some values of $\underline{k}$. From this graph it should be clear that the largest fixed point is decreasing in the costs and that the number of fixed points go from 1 to 3 and then back to 1 as the costs of investment in human capital increase.

5.4 Continuation Equilibria in the Model with Trade

We now assume that two equal sized countries, h (home) and f (foreign) trade goods without costs, while labor is immobile.

The number of potential forms of continuation equilibria now swells to 9 different cases (of which few can be ruled out) and to reduce the number of possible equilibria we specialize the model and

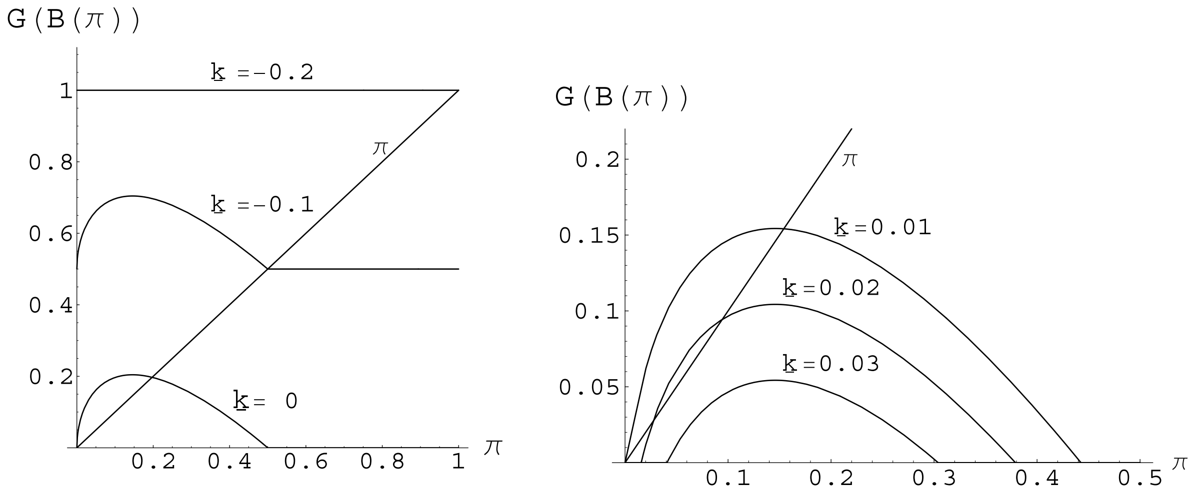

Figure 6: Equilibrium fixed point maps for several values of $\underline{k}$, with $\eta = 2/3, \alpha = 1/2$

set $\eta = 2/3$ and $\alpha = 1/2$ (meaning that (44) is the relevant fixed point equation in the autarky model). With these parameter values the continuation equilibrium can be of three different forms:

Equilibria of type A^t: all workers in country h are employed in sector 2. In country f, all workers with signal g and a fraction $\beta \in (0,1)$ of the workers with signal b are in sector 1, while the remaining $(1-\beta)$ workers with signal b are in sector 2. For this to be consistent with equilibrium it must be that $w_g^h = w_b^h = 1$ and

$$w_g^f = p_1 \frac{2\pi^f}{1+\pi^f} \qquad w_b^f = 1 = p_1 \frac{\pi^f}{2-\pi^f}, \tag{46}$$

so the equilibrium price is determined as $p_1 = \frac{2-\pi^f}{\pi^f}$ from the condition that b workers in f must be equally valuable in both sectors. To check that this is an equilibrium we must show that there exists $\beta \in (0,1)$ such that the goods market clear given this price and also that there are no incentives to use workers with signal g in the high tech sector in country h. These conditions are satisfied whenever $\pi^f \geq 4\pi^h/(1+3\pi^h)$.

Equilibria of type B^t: a fraction $\gamma \in (0,1)$ of the workers with signal g in country h and all workers with signal g in f are in the high tech sector. The remaining workers are in the low tech sector. For this to be consistent with equilibrium it must be that $w_b^h = w_b^f = 1$ and that

$$w_g^f = p_1 \frac{2\pi^f}{1+\pi^f} \qquad w_g^h = 1 = p_1 \frac{2\pi^h}{1+\pi^h}. \tag{47}$$

Hence, $p_1 = \frac{1+\pi^h}{2\pi^h}$ follows from the condition that workers with signal g in country h must be equally valuable in each sector. Again, we must check that there exists $\gamma \in (0,1)$ such that market clearing is satisfied given this equilibrium price and that there are no incentives to use b workers in sector 1 in country f. These conditions are satisfied whenever $\pi^h(3-2\pi^h)/(1+2\pi^h) \le \pi^f \le 4\pi^h/(1+3\pi^h)$.

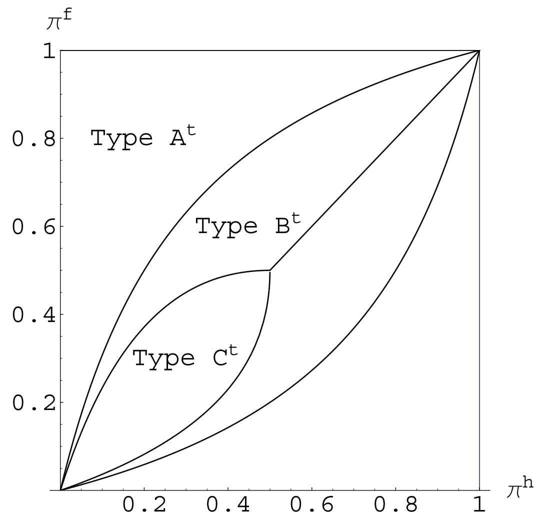

Figure 7: Types of asymmetric equilibria, with $\eta = 2/3, \alpha = 1/2$

Equilibria of type C^t: in both countries, workers with signal g are employed in sector 1 and workers with signal b are employed in sector 2. This requires that $w_b^h = w_b^f = 1$ and that

$$w_g^f = p_1 \frac{2\pi^f}{1+\pi^f} \qquad w_g^f = p_1 \frac{2\pi^h}{1+\pi^h}. \tag{48}$$

In this case the equilibrium price (rather than the randomizing probabilities) are determined from the goods market clearing condition, that is p_1 now solves

$$p_1 = \frac{x_2}{x_1} = \frac{4-\pi^f-\pi^h}{2(\pi^f+\pi^h)}. \tag{49}$$

To check that this is an equilibrium we must make sure that there are no incentives to reallocate workers given these prices and wages. Since $\pi^h \le \pi^f$ by labeling of the countries, the relevant conditions are that there are no incentives to use workers with signal g in sector 2 in h, which simply requires that $w_g^h \ge 1$, and that there are no incentives to use workers with signal b in sector 1 in country f. The range for this type of equilibrium is when $\pi^h \le \pi^f \le \pi^h(3-2\pi^h)/(1+2\pi^h)$.

Figure 7 shows the different regions of investment behavior that is relevant for each type of equilibrium. Detailed derivations are in Moro and Norman [11].

5.5 Equilibria with Specialization and Trade

Substituting the wages and prices from Section 5.4 into the expression for benefits of investment (35) we obtain closed form expressions for benefits of investment in each country as a function of (π^f, π^h) given by

$$B^f\left(\pi^h, \pi^f\right) = \begin{cases} \frac{1}{6}\sqrt{\frac{\pi^f}{2-\pi^f}}\left(\frac{2-\pi^f}{\pi^f}\frac{2\pi^f}{1+\pi^f} - 1\right) & \text{if } \pi^f \geq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{1}{6}\sqrt{\frac{2\pi^h}{1+\pi^h}}\left(\frac{1+\pi^h}{2\pi^h}\frac{2\pi^f}{1+\pi^f} - 1\right) & \text{if } \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \leq \pi^f \leq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{1}{6}\sqrt{\frac{2\left(\pi^h+\pi^f\right)}{4-\pi^h-\pi^f}}\left(\frac{4-\pi^h-\pi^f}{2\left(\pi^h+\pi^f\right)}\frac{2\pi^f}{1+\pi^f} - 1\right) & \begin{array}{c}\pi^f \leq \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \text{ and} \\ \pi^h \leq \frac{\pi^f(3-2\pi^f)}{(1+2\pi^f)}\end{array} \\ 0 & \text{otherwise} \end{cases}, \tag{50}$$

and symmetrically for country h. Note that for $\pi^h = \pi^f = \pi$ this simplifies to

$$B^f(\pi, \pi) = \max\left\{\frac{1}{6}\sqrt{\frac{2\pi}{2-\pi}}\left(\frac{2-\pi}{2\pi}\frac{2\pi}{1+\pi} - 1\right), 0\right\} \tag{51}$$

and is equivalent to the associated benefits of investment under autarky computed according to (35). Intuition can be gained by simply comparing (50) and (51). If we reduce π^h in (50) the price of good 1 will increase monotonically from $\frac{2-\pi^f}{2\pi^f}$ (the price when $\pi^h = \pi^f$) to $\frac{2-\pi^f}{\pi^f}$ (the price when $\pi^f \geq \frac{4\pi^h}{(1+3\pi^h)}$. If we express the incentives to invest in f in terms of the price and the Bayes rule probability of investment conditional on the good signal we obtain $\frac{1}{6}\left(\sqrt{p_1}P\left(g, \pi^f\right) - \frac{1}{\sqrt{p_1}}\right)$; this expression shows that, for a given π^f, incentives are monotonically increasing in p_1. Hence, lower investments in h increases incentives in f and higher investments in f lowers incentives in h. These "negative cross effects" tend to create asymmetric equilibria also when the conditions for Proposition 8 are satisfied, so countries may endogenously chose to specialize as rich and poor countries.

To show that asymmetric equilibria are possible for a large set of parameters, we have considered the same class of cost distributions used for the numerical exercise performed in the autarky case (that is, we assumed a uniform distribution of cost k on $[\underline{k}, \underline{k} + 0.2]$. The easiest way to look for asymmetric equilibria is to check whether there are equilibria where country h has zero incentives to invest and consequently their (candidate) equilibrium investment can be immediately pinned down at $\pi^h = \max[-5 \cdot \underline{k}, 0]$. Such equilibria must necessarily be of either type A^t or B^t because in equilibria of type C^t workers in country h have positive incentives to invest. The problem of verifying whether there is a level of investment in country f that supports an asymmetric equilibrium corresponds to finding the solution of a fixed point equation in one variable.

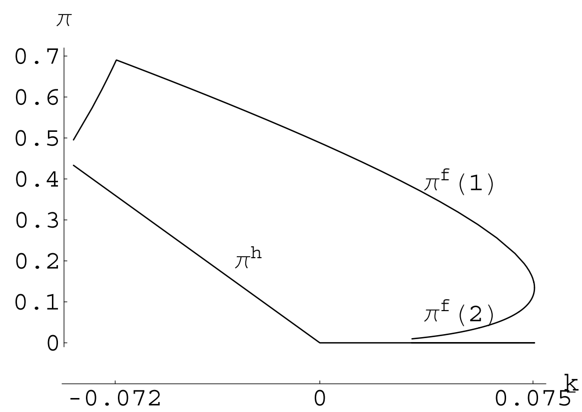

Figure 8: Equilibrium investments under trade with $\eta = 2/3, \alpha = 1/2$

We have computed that such equilibria exist provided that $\underline{k}$ is not too small or too big. In Figure 8 the curve labeled π^h displays equilibrium levels for country h for different values of k in the relevant range. For k approximately between -0.086 and -0.072 there is an asymmetric equilibrium where a fraction of g workers in country h produce the high tech good (equilibria of type B^t). In there range for k approximately between -0.072 and 0.075 there is an equilibrium where h is fully specialized in production of the low tech good (equilibria of type A^t, see curve labeled $\pi^f(1)$). Finally, for k between 0 and 0.075 a second type A^t equilibrium appears where country f investment is small but positive (see curve labeled $\pi^f(2)$).

5.5.1 Example 1: Trade May be Beneficial Only to the Rich Country

We use the numerical computations to show that there are regions of the parameter space where trade makes citizen of the rich country better off and citizens of the poor country worse off relative to autarky. In Table 1 we compare the unique equilibrium under autarky with one of the asymmetric trade equilibria when k is uniformly distributed on $[-0.02, 0.18]$. Quantities of aggregate production and consumption under trade are computed considering aggregate population size equal to 1 in each country.

In this case there is a unique autarky equilibrium with investment $\pi = .269$ and a unique trade equilibrium with investment equal to 0.1 and 0.548. Lower investment under trade hurts country

$\eta = \frac{2}{3}$, $\alpha = \frac{1}{2}$, $k \sim U[-0.02, , 0.18]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0.1$	$\pi^f = 0.548$	$\pi = .269$
Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = .463$ $y_2^f = .226$	$y_1 = 0.179$ $y_2 = .577$
Consumption	$x_1^h = .189$ $x_2^h = .5$	$x_1^f = .274$ $x_2^f = .726$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.0897$	$B(\pi) = 0.0338$
Gross expected utility	$u^h = .307$	$u^f = .446$	$u = .321$
Expected utility net of inv. cost	$u^h = .308$	$u^f = .427$	$u = .319$
Expected utility if invest	$E(u^h\|\text{inv}) = .307 - k$	$E(u^h\|\text{inv}) = .487 - k$	$E(u^h\|\text{inv}) = .346 - k$
Expected utility if don't invest	$E(u^h\|\text{no inv}) = .307$	$E(u^h\|\text{no inv}) = .397$	$E(u^h\|\text{no inv}) = .313$
Prices	$p_1 = 2.648$ $w_g^h = w_b^h = 1$ $w_g^f = 1.875$ $E(w_f) = 1.452$		$p_1 = 3.216$ $w_b = 1$ $w_g = 1.364$ $E(w) = 1.154$

Table 1: Features of the trade and autarky equilibria in Example 1

h citizens and benefits country f citizens in terms of average welfare. This is true not only in aggregate terms, but also from a Pareto comparison: inspection of the expected utility of investors and not investors reveal that each worker in country h is better off under autarky and each worker in country f better off in the trade equilibrium where they invest more.

5.5.2 Example 2: Trade May Make Both Countries Better Off

We now consider an example where trade makes both countries better off. For maximal simplicity we rig this example so that the "free rider problem" in human capital investments is so severe the unique equilibrium under autarky is the trivial equilibrium. However, with trade, the existence of the other country means that, for any investment π^f in country f, the price of good 1 is higher than without trade under the assumption that there are no investments in the other country. Hence, trade potentially allows for new markets to emerge that would not operate without trade.

In Table 2 we summarize one such example where the market for good 1 can only operate with international trade. Here, there are actually multiple trade equilibria and the numbers in the table is for the equilibrium with the largest fraction of investors in the country producing good 1[8].

$\eta=\frac{2}{3}$, $\alpha=\frac{1}{2}$, $k\sim U[.04,.16]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h=0$	$\pi^f=0.353$	$\pi=0$
Production	$y_1^h=0$ $y_2^h=1$	$y_1^f=.284$ $y_2^f=.323$	$y_1=0$ $y_2=1$
Consumption	$x_1^h=.107$ $x_2^h=.5$	$x_1^f=.177$ $x_2^f=.823$	$x_1=y_1$ $x_2=y_2$
Gross incentives to invest	$B^h(\pi^h,\pi^f)=0$	$B^f(\pi^h,\pi^f)=0.1107$	$B(\pi)=0$
Gross average utility	$u^h=.232$	$u^f=.381$	$u=0$
Avg. utility net of inv. cost	$u^h=.232$	$u^f=.355$	$u=0$
Expected utility if invest	$E(u^h\|\text{inv})=.232-k$	$E(u^h\|\text{inv})=.452-k$	$E(u^h\|\text{inv})=0-k$
Expected utility if don't invest	$E(u^h\|\text{no inv})=.232$	$E(u^h\|\text{no inv})=.342$	$E(u^h\|\text{no inv})=0$
Prices	$p_1=4.660$ $w_g^h=w_b^h=1$ $w_g^f=2.433$ $E(w_f)=1.647$		$p_1=-$ $w_b=1$ $w_g=-$ $E(w)=1$

Table 2: Features of the trade and autarky equilibria in Example 2

Consumers are always happier when consuming both goods than when only consuming one good, so it follows immediately from the fact that the new market opens up that trade is beneficial for both countries. More elaborate examples where trade improves the welfare for both countries in spite of both goods being consumed initially can also be constructed.

5.5.3 Example 3: An Asymmetric Equilibrium with Trade May Be the Only Stable Outcome

It may be argued that a weakness with the examples above is that, no matter what, there is always a symmetric equilibrium without trade. However, for many parametrizations of the model the symmetric equilibrium is "destabilized" by when the economy is opened up for international trade.

Since our model lacks real time "stability" is obviously an ad hoc criterion that corresponds to the seemingly myopic adjustment dynamic where $\pi^j_{t+1}=G(B^j(\pi^j_t,\pi^k_t))$, $j,k=h,f$, $j\neq k$ (or the

[8]There is also an equilibrium with $\pi^h=0$, $\pi^f=0.0157$. However, unlike the equilibrium in Table 2 this is unstable in an ad hoc static sense.

natural continuous analogue)[9].

Assume that $\underline{k} < 0$, so that there is a unique autarky equilibrium, which we denote by π^A. It is then immediate that π^A must be stable since $G(B(\pi))$ must intersect the 45 degree line from above. It also follows immediately that $(\pi^h, \pi^f) = (\pi^A, \pi^A)$ is an equilibrium also when the countries are allowed to trade.

We want to analyze the effects of small deviations from the symmetric equilibrium (π^A, π^A). Consider the change in relative price first. Let $p(\pi^h, \pi^f)$ denote the unique equilibrium price when fractions $\pi = (\pi^h, \pi^f)$ invest (see 49). When $\pi^h = \pi^f = \pi$ the price of good 1 is $p(\pi, \pi) = (4 - \pi - \pi)/2(\pi + \pi) = (2 - \pi)/\pi$ which is exactly the price under autarky. Hence

$$\begin{aligned} \frac{d}{d\pi} p(\pi, \pi) &= \frac{-1}{(\pi)^2} \qquad \text{(relevant under autarky)} \\ \frac{\partial}{\partial \pi^f} p(\pi^h, \pi^f) &= \frac{-2}{(\pi^h + \pi^f)^2} \qquad \text{(relevant with trade)} \end{aligned} \tag{52}$$

and evaluating each expression at (π^A, π^A) we have that

$$\left. \frac{d}{d\pi} p(\pi, \pi) \right|_{\pi = \pi^A} - \left. \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f} \right|_{\pi^h = \pi^f = \pi^A} = \frac{-1}{(\pi^A)^2} - \frac{-2}{4(\pi^A)^2} = \frac{-1}{2(\pi^A)^2} < 0 \; . \tag{53}$$

Hence, the effect on the price is more negative in the autarky model.

Incentives to invest for a worker in country f can be expressed as

$$B^f(\pi^h, \pi^f) = \frac{1}{6} \left(\sqrt{p(\pi^h, \pi^f)} P\left(g, \pi^f\right) - \frac{1}{\sqrt{p(\pi^h, \pi^f)}} \right). \tag{54}$$

Recall that if $\pi^h = \pi^f = \pi$ then the incentives to invest in country j expressed by $B^j(\pi, \pi)$ are equivalent to the incentives to invest in autarky computed according to (35). Now, if we differentiate (54) with respect to π^f for the autarky model (where the two arguments of B^f are restricted to be equal - see (51)) and trade model (where the arguments are unrestricted) and evaluate at the autarky (symmetric) equilibrium we get respectively

$$\begin{aligned} \left. \frac{dB(\pi, \pi)}{d\pi} \right|_{\pi = \pi^A} &= \underbrace{\frac{1}{6} \sqrt{p^A} \left. \frac{dP(g, \pi)}{d\pi} \right|_{\pi = \pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{\sqrt{p^A}} \left(P\left(g, \pi^A\right) + \frac{1}{2p^A} \right) \left. \frac{dp(\pi, \pi)}{d\pi} \right|_{\pi = \pi^A}}_{\text{“price effect”}} \\ \left. \frac{\partial B^f(\pi^h, \pi^f)}{\partial \pi^f} \right|_{\pi^h = \pi^f = \pi^A} &= \underbrace{\frac{1}{6} \sqrt{p^A} \left. \frac{dP(g, \pi^f)}{d\pi^f} \right|_{\pi^f = \pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{\sqrt{p^A}} \left(P\left(g, \pi^A\right) + \frac{1}{2p^A} \right) \left. \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f} \right|_{\pi = \pi^A}}_{\text{“price effect”}} . \end{aligned} \tag{55}$$

[9] It is however possible to get a dynamic system like this without cheating if one is willing to assume that employers can not differentiate between workers of different cohorts.

Hence, in each case we can decompose the effect on the incentives as a positive "information effect" and a negative "price effect". The information effect is the same in the two expressions. However by virtue of (52) we know that the price effect is more negative under autarky so we conclude that the slope of $B^f\left(\pi^f, \pi^h = \pi^A\right)$ must be larger than the slope of the autarky benefits of investment $B(\pi)$ when both functions are evaluated at π^A. Hence, it is possible that $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersects the 45 degree line from below at $\pi^f = \pi^A$ even though $G(B(\pi))$ intersects from above. Since the curve $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersecting the 45 degree line from below is a *sufficient condition for local instability* this shows that we can construct such examples.

We have verified numerically that we do not need to look hard for examples where the autarky equilibrium is unstable. One example that works is when k is uniformly distributed on $[0, 2]$. The unique autarky equilibrium is then $\pi = .0067$. Figure 9 illustrates that $G(B(\pi))$ cuts the 45^0 line from above (indicating stability of the autarky equilibrium) while $G(B^f\left(\pi^f, \pi^h = 0.067\right))$ cuts the 45^0 line from below (the derivative is $\partial G(B^f(\pi^f, \pi^h))/\pi^f = 1.244$) indicating that the symmetric equilibrium under trade is unstable. We have computed that under this parametrization there is a stable asymmetric equilibrium with $\pi^f = .0283$, $\pi^h = 0$.

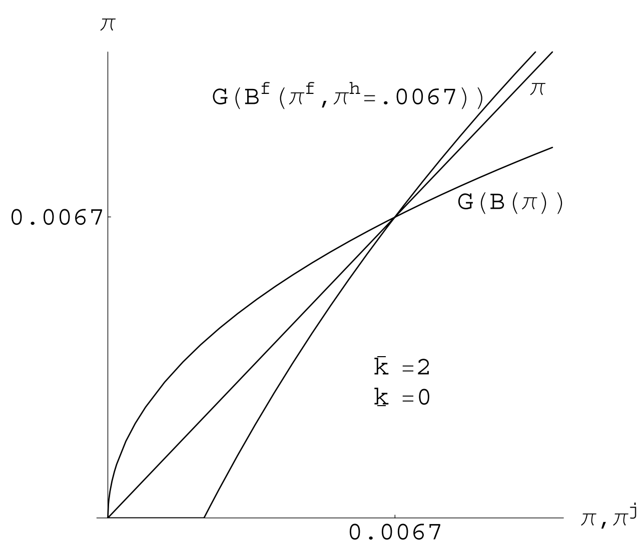

Figure 9: Best responses under trade and autarky, at the autarky equilibrium

5.5.4 Interpretation of the Example-Economics of Specialization

The crucial point of the examples is that trade allows countries to specialize in equilibrium. Despite the countries being intrinsically identical, such specialization may (but need not be) beneficial to all.

In a sense, the specialization may be viewed as an imperfect "solution" to the informational problem in the model[10]. Under much more general circumstances than in the example it can be shown that the production possibilities set expands if the differences in investment behavior is increased, but the total quantity of investors is held constant.

A simple intuition for the increase in world production is that specialization reduces the number of "mistakes" in how workers are matched to jobs: in autarky, a fair number of workers are working in sector 1 although they are completely useless in that sector. This inefficiency is reduced in the equilibrium with specialization.

6 Discussion

6.1 Our Contribution to the Literature

Our model is a perfectly competitive model where workers are better informed about their capabilities than the firms. Previously, Gene Grossman and Giovanni Maggi have both considered the implications of asymmetric information on trade in several papers. In particular, Grossman and Maggi [5] consider an essentially competitive environment with imperfect observability of talent. However, their model is aimed at fundamentally different issues and for their purposes it is sufficient to consider how trade is affected by exogenous differences in talent distributions. Moreover their model assume complementarities in production, while our model can generate specialization also when such complementarities are assumed away (as in the examples in section 5).

Many properties of our model are shared with models in the increasing returns literature. The result that we may get specialization even when there is a unique equilibrium in autarky can be obtained in several other models, for example as in Either [4]. We nevertheless find it a useful observation that this sort of logic can be used to rationalize endogenous differences in factor resources.

[10] For a detailed elaboration on this point in the context of discrimination, see Norman [12].

We also think that a very appealing property of the model is that the externality is derived rather than postulated. This yields additional restrictions from the theory that we will explore further in future research. Models of economic geography/agglomeration (Krugman [6], Puga and Venables [16] and others) also derive an "externality" from primitive assumptions about transportation costs and factor mobility (although this "pecuniary" externality is what we refer to as "price effects"). Indeed, in a mechanical sense, our model works much like these models. We get action from feedbacks between price effects and the local informational externality, while that literature get it from the interplay between the pecuniary externality and increasing returns to scale.

In spite of the similarities, our model and the economic geography models are models based on fundamentally different premises and address orthogonal issues. For example, the geography models need labor mobility to get any clustering of firms and are therefore good models to think about differences within regions with small barriers to labor mobility, a natural model for the European Union or regional differences in the Us. Our model takes the opposite perspective by ruling out labor mobility completely and seems more appropriate for thinking about North-South differences in development and trade patterns.

Finally, we find it a virtue that our model is very close to the standard Hecksher-Ohlin-Samuelson setup. Since the model has all the usual factor content implications (but in terms of labor with different skills) as well as implications on how wage *distributions* should relate to economic development and the skill content of trade it is our hope that a quantitative assessment of an appropriately enriched version of our model will be relatively straightforward.

6.2 Topics for Further Research

There are several natural ways to extend the model and in future research we intend to do so in order to make the model more suitable for addressing economic development issues. We believe that introducing an observable component to human capital is an important step for judging whether the type of explanation for differences in economic development given in this paper can be quantitatively important or not. This is because returns to observable human capital investments have been studied extensively in the empirical literature, so a natural reality check of the model is to see whether it can produce wage distributions similar to those observed in the real world when there is both observable and unobservable human capital.

Another extension that may be interesting for quantitative purposes is to introduce capital, land or both these factors into the model. While the introduction of capital will lead to relatively minor changes for the theory as long as it is mobile, the hope is that this extension may make it possible to calibrate the parameters in the production technology more easily by using arbitrage conditions together with actual factor rewards.

A Appendix: Proofs

Proof of Lemma 1. To conserve space we define

$$
\begin{aligned}
f_\pi(\theta) &= \pi f_q(\theta) + (1-\pi) f_u(\theta) \\
F_\pi(\theta) &= \pi F_q(\theta) + (1-\pi) F_u(\theta)
\end{aligned} \tag{A1}
$$

Note that f_π (F_π) is a valid probability density (cumulative) that inherits all smoothness properties from f_q and f_u. Given any feasible labor demand $l : \Theta \to R_+^4$ (that is, l must be integrable and satisfy (7)), define $t(\theta) \equiv \frac{l_1^c(\theta)+l_2^c(\theta)}{f_\pi(\theta)}$, the fraction of workers with signal θ that is employed in the complex task. Obviously, $0 \leq t(\theta) \leq 1$ for feasibility efficiency requires that $\sum_i \sum_t l_i^t(\theta) = f_\pi(\theta)$ for almost all θ for each $l : \Theta \to R_+^4$ that generates factor inputs z on the efficiency frontier of $Z(\pi)$. Thus, for each z on the frontier of $Z(\pi)$ there exists some $t : [0,1] \to [0,1]$ such that

$$
\begin{aligned}
c_1 + c_2 &= \int t(\theta) f_\pi(\theta) P(\theta,\pi) d\theta = \int t(\theta) \pi f_q(\theta) d\theta \\
s_1 + s_2 &= \int (1-t(\theta)) f_\pi(\theta) d\theta.
\end{aligned} \tag{A2}
$$

Claim For each z on the efficiency frontier of $Z(\pi)$ there exists some $\theta^* \in [0,1]$ such that

$$
\begin{aligned}
c_1 + c_2 &= \pi(1 - F_q(\theta^*)) \\
s_1 + s_2 &= \pi F_q(\theta^*) + (1-\pi) F_u(\theta^*)
\end{aligned} \tag{A3}
$$

Proof. For contradiction, suppose that there is a point $z' = (c_1', s_1', c_2', s_2')$ on the efficiency frontier of $Z(\pi)$ and some $\theta' \in \Theta$ such that $\int_0^{\theta'} \pi t(\theta) f_q(\theta) d\theta > 0$ and $\int_{\theta'}^1 (1-t(\theta)) f_\pi(\theta) d\theta > 0$. By choice of θ' we may without loss assume that

$$
\int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta = \int_{\theta'}^1 (1-t(\theta)) f_\pi(\theta) d\theta > 0 \tag{A4}
$$

Consider some alternative labor demand that generates some $\widehat{t}$, where $\widehat{t}(\theta) = 0$ for $\theta \leq \theta'$ and $\widehat{t}(\theta) = 1$ for $\theta > \theta'$ and let $z'' = (c_1'', s_1'', c_2'', s_2'')$ denote some feasible inputs that satisfies $c_1'' + c_2'' = \int \pi \widehat{t}(\theta) f_q(\theta) d\theta$ and $s_1'' + s_2'' = \int (1 - \widehat{t}(\theta)) f_\pi(\theta) d\theta$. Then,

$$
\begin{aligned}
s_1' + s_2' \underset{\text{/By (A2)/}}{=} \int_0^{\theta'} (1 - t(\theta)) f_\pi(\theta) d\theta + \int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta &= \\
\underset{\text{/By (A4)/}}{=} \int_0^{\theta'} (1 - t(\theta)) f_\pi(\theta) d\theta + \int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta &= \int_0^{\theta'} f_\pi(\theta) d\theta = \\
\underset{\text{/Construction of } \widehat{t}\text{/}}{=} \int_{\theta \in [0,1]} (1 - \widehat{t}(\theta)) f_\pi(\theta) d\theta - \int_{\theta'}^{\theta'} f_\pi(\theta) d\theta &= s_1'' + s_2'',
\end{aligned} \tag{A5}
$$

The input of labor in the complex task in the original plan is

$$
\begin{aligned}
c_1' + c_2' \underset{\text{/By (A2)/}}{=} \int_0^{\theta'} t(\theta) p(\theta, \pi) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta &< \\
\underset{/P(\cdot,\pi) \text{ increasing/}}{<} P(\theta', \pi) \int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta &= \\
\underset{\text{/By (A4)/}}{=} P(\theta', \pi) \int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta &\underset{/P(\cdot,\pi) \text{ increasing/}}{<} \int_{\theta'}^1 \pi f_q(\theta) d\theta \\
= \int_{\theta \in [0,1]} \pi \widehat{t}(\theta) f_q(\theta) d\theta &= c_1'' + c_2'',
\end{aligned} \tag{A6}
$$

so $c_1'' + c_2'' > c_1' + c_2'$ and $s_1'' + s_2'' = s_1' + s_2'$ it follows that the input in the complex task can be increased without decreasing the input in the simple task. It is then straightforward (argument is omitted) to use continuity of the rule $\widehat{t}$ to verify that it is then also possible to increase the input of labor in both tasks and since labor can be split arbitrarily across industries it follows that inputs in both jobs and both industries can be increased relative the hypothetical point on the frontier ■

Since any point on the efficiency frontier satisfies (A3) for some $\theta^* \in [0,1]$ we may eliminate θ^* for a z on the frontier and conclude that

$$
\begin{aligned}
s_1 + s_2 &= \pi F_q(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)) + (1 - \pi) F_u(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)) = \\
&= \pi - c_1 - c_2 + (1 - \pi) F_u(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)),
\end{aligned} \tag{A7}
$$

so $g(c_1 + c_2, s_1 + s_2; \pi) = 0$ for g defined in (11). It is easy to verify that $g(c_1 + c_2, s_1 + s_2; \pi) > 0$ for interior and $g(c_1 + c_2, s_1 + s_2; \pi) < 0$ for non-feasible points, completing the proof. ■

Proof of Lemma 2. We define $H(c) = F_u\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)$ and apply the inverse function theorem to conclude that $H'(c) = -\frac{f_u\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)}{f_q\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)}$. The ratio $\frac{f_u(\theta)}{f_q(\theta)}$ is the inverse of the likelihood ratio, which is strictly decreasing, F_q^{-1} is strictly increasing $\frac{\pi-c}{\pi}$ is strictly decreasing, so we conclude that $H'(c)$ is strictly decreasing, that is H is strictly concave in c. Now take a pair of inputs $(c', s'), (c'', s'')$ and let (c^λ, s^λ) denote a convex combination given any $\lambda \in (0,1)$. If $c' \neq c''$ we have that,

$$\begin{aligned} g\left(c^\lambda, s^\lambda; \pi\right) &= \pi - c^\lambda - s^\lambda + H\left(c^\lambda\right) = \\ &= \pi - c^\lambda - s^\lambda + H\left(c^\lambda\right) + \lambda H(c') - \lambda H(c') + (1-\lambda) H(c'') - (1-\lambda) H(c'') = \\ &= \lambda g(c', s'; \pi) + (1-\lambda) g(c'', s'', \pi) + H\left(c^\lambda\right) - \lambda H(c') - (1-\lambda) H(c'') > \\ &> \lambda g(c', s'; \pi) + (1-\lambda) g(c'', s'', \pi) > \min\{g(c', s'; \pi), g(c'', s'', \pi)\} \end{aligned} \tag{A8}$$

since H is strictly concave. If $c' = c''$, then $s' \neq s'$ and if we without loss assume $s' < s''$, then $g(c', s'; \pi) - g(c'', s'', \pi) = s'' - s' > 0$, implying that

$$\begin{aligned} g\left(c^\lambda, s^\lambda; \pi\right) &= g(c'', s'', \pi) + \lambda\left(g(c', s'; \pi) - g(c'', s'', \pi)\right) > \\ &> g(c'', s'', \pi) = \min\{g(c', s'; \pi), g(c'', s'', \pi)\}. \end{aligned} \tag{A9}$$

The two cases $c' \neq c''$ and $c' = c''$ exhaust the possibilities and the result follows. ■

Proof of Lemma 3. Let $Z^*(\pi) = \left\{c, s \in R_+^2 | g(c, s; \pi) \geq 0\right\}$ be the set of feasible aggregate factor inputs. It follows immediately from the (strict) quasi-concavity of g that $Z^*(\pi)$ is convex. Convexity of $Z(\pi)$ is then rather clear and can be established in a number of different ways. Easiest is to note that if there exists $z, z' \in Z(\pi)$ and $\lambda \in [0,1]$ such that $\lambda z + (1-\lambda) z' \notin Z(\pi)$, then we may let (c, s) and (c', s') represent the aggregate factor inputs corresponding to z and z'. Clearly, $\lambda z + (1-\lambda) z' \notin Z(\pi)$ means that $g(\lambda c + (1-\lambda) c', \lambda s + (1-\lambda) s'; \pi) < 0$, which violates the convexity of $Z^*(\pi)$. ■

Proof of Lemma 4. Let $x, x' \in X(\pi)$ be two plans with associated factor inputs $z, z' \in Z(\pi)$. Since $Z(\pi)$ is convex, any convex combination $z^\lambda = \lambda z + (1-\lambda) z \in Z(\pi)$ and,

$$y^i\left(c_i^\lambda, s_i^\lambda\right) \geq \lambda y^i(c_i, s_i) + (1-\lambda) y^i(c_i', s_i') = \lambda x_i + (1-\lambda) x_i', \tag{A10}$$

for $i = 1, 2$. Hence $x^\lambda = \lambda x + (1-\lambda) x' \in X(\pi)$. ■

Proof of Lemma 5. Since $X(\pi)$ is weakly convex (Lemma 4) a failure of strict convexity implies that there is a linear segment on the frontier of $X(\pi)$. The frontier of $X(\pi)$ may be described as a function $x_1(x_2)$, which for each $x_2 \geq 0$ (and less than the maximal production of good 2) is given by the solution to the problem

$$\begin{aligned} x_1(x_2) &= \max_{c_1,c_2,s_1 s_2} y^1(c_1,s_1) \\ \text{s.t } y^2(c_2,s_2) &\geq x^2, g(c_1+c_2,s_1+s_2;\pi) \geq 0. \end{aligned} \tag{A11}$$

Now, if there is a linear segment on the frontier of $X(\pi)$, then there exists some interval $[x_2', x_2'']$ and a constant k such that $\frac{dx_1(x_2)}{dx_2} = k$ for all $x_2 \in [x_2', x_2'']$. By a straightforward application of the envelope theorem if follows that the effect of a small increase in x_2 on the maximal choice of x_1 equals the negative of the multiplier on the first constraint and from the first order conditions to (A11) we find that

$$\frac{dx_1(x_2)}{dx_2} = -\frac{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial c}}{\frac{\partial y^1(c_2(x_2),s_2(x_2))}{\partial c}} = -\frac{\frac{\partial y^2(c_1(x_2),s_1(x_2))}{\partial s}}{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial s}}. \tag{A12}$$

Combining (A12) with the fact that $\frac{dx_1(x_2)}{dx_2}$ is constant on $[x_2', x_2'']$ it follows that for any $x_2 \in [x_2', x_2'']$ we have that

$$\frac{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial c}}{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial s}} = \frac{\frac{\partial y^1(c_1(x_2'),s_1(x_2''))}{\partial c}}{\frac{\partial y^1(c_1(x_2'),s_1(x_2'))}{\partial s}} \text{ and } \frac{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial c}}{\frac{\partial y^1(c_2(x_2),s_2(x_2))}{\partial s}} = \frac{\frac{\partial y^2(c_2(x_2'),s_2(x_2'))}{\partial c}}{\frac{\partial y^1(c_2(x_2'),s_2(x_2'))}{\partial s}}, \tag{A13}$$

which by strict quasi-concavity of y^1 and y^2 implies that $\frac{c_1(x_2)}{s_1(x_2)} = \frac{c_1(x_2')}{s_1(x_2')}$ and $\frac{c_2(x_2)}{s_2(x_2)} = \frac{c_2(x_2')}{s_2(x_2')}$ for all $x_2 \in [x_2', x_2'']$. That is, when moving across the linear segment of $x_1(x_2)$ it must be that the factor ratios are constant for the associated production plans. Since one sector is more intensive in the complex labor and since we are transferring resources between the two sectors while moving along the frontier, this leads to a contradiction. To see this, let $x_2^\lambda = \lambda x_2' + (1-\lambda) x_2''$ for any $\lambda \in [0,1]$. Also, define r_i to be the (in the segment) constant factor ratio in sector $i = 1, 2$, so that $r_i \equiv \frac{c_i(x_2')}{s_i(x_2')} = \frac{c_i(x_2^\lambda)}{s_i(x_2^\lambda)}$ for any $\lambda \in [0,1]$. Note that since sector 1 is more intensive in c than is sector 2 (A13) implies that $r_1 > r_2$. Linearity of $x_1(x_2)$ means that

$$x_1\left(x_2^\lambda\right) = \lambda x_1\left(x_2'\right) + (1-\lambda) x_1\left(x_2''\right) \tag{A14}$$

and using (A14), the fact that the factor ratios are convex, and properties of constant returns technologies we than have that (where we employ the convention that $x_2(x_2) = x_2$),

$$\begin{aligned} s_i(x_2^\lambda) y^i(r_i,1) &= y^i(c_i(x_2^\lambda), s_i(x_2^\lambda)) \equiv x_i(x_2^\lambda) = \lambda x_i\left(x_2'\right) + (1-\lambda) x_i\left(x_2''\right) = \\ &= \lambda y^i\left(c_i(x_2'), s_i(x_2')\right) + (1-\lambda) y^i\left(c_i(x_2''), s_i(x_2'')\right) = \\ &= \left[\lambda s_i\left(x_2'\right) + (1-\lambda) s_i\left(x_2''\right)\right] y^i(r_i,1) \end{aligned}$$

for sectors $i = 1, 2$. Hence, $s_i\left(x_2^\lambda\right) = \lambda s_i\left(x_2'\right) + (1-\lambda)\, s_i\left(x_2''\right)$ and since $c_i\left(x_2\right) = r_i s_i\left(x_2\right)$ for $x_2 \in \left[x_2', x_2''\right]$ it also follows that $c_i\left(x_2^\lambda\right) = \lambda c_i\left(x_2'\right) + (1-\lambda)\, c_i\left(x_2''\right).$ Now let $c\left(x_2\right) \equiv c_1\left(x_2\right) + c_2\left(x_2\right)$ and $s\left(x_2\right) \equiv s_1\left(x_2\right) + s_2\left(x_2\right)$ and observe that

$$(c(x_2^\lambda), s(x_2^\lambda)) = \left(\lambda c\left(x_2'\right) + (1-\lambda)\, c\left(x_2''\right), \lambda s\left(x_2'\right) + (1-\lambda)\, s\left(x_2''\right)\right). \tag{A15}$$

Moreover, using the constant factor ratio and constant returns to scale we find that

$$\begin{aligned} c_1\left(x_2''\right) - c_1\left(x_2'\right) &= \frac{r_1\left(x_1\left(x_2''\right) - x_1\left(x_2'\right)\right)}{y^1\left(r_1, 1\right)} \text{ and } c_2\left(x_2''\right) - c_2\left(x_2'\right) = \frac{r_2\left(x_2'' - x_2'\right)}{y^2\left(r_2, 1\right)} \\ s_1\left(x_2''\right) - s_1\left(x_2'\right) &= \frac{\left(x_1\left(x_2''\right) - x_1\left(x_2'\right)\right)}{y^1\left(r_1, 1\right)} \text{ and } s_2\left(x_2''\right) - s_2\left(x_2'\right) = \frac{\left(x_2'' - x_2'\right)}{y^2\left(r_2, 1\right)}, \end{aligned} \tag{A16}$$

so if $c\left(x_2''\right) \geq c\left(x_2'\right)$, then

$$\begin{aligned} 0 &\leq \frac{r_1\left(x_1\left(x_2''\right) - x_1\left(x_2'\right)\right)}{y^1\left(r_1, 1\right)} + \frac{r_2\left(x_2'' - x_2'\right)}{y^2\left(r_2, 1\right)} \Rightarrow /\left(x_2'' - x_2'\right) > 0 \text{ and } r_1 > r_2/ \\ 0 &< r_1\left(\frac{\left(x_1\left(x_2''\right) - x_1\left(x_2'\right)\right)}{y^1\left(r_1, 1\right)} + \frac{\left(x_2'' - x_2'\right)}{y^2\left(r_2, 1\right)}\right) = r_1\left(s_1\left(x_2''\right) - s_1\left(x_2'\right) + s_2\left(x_2''\right) - s_2\left(x_2'\right)\right) \end{aligned} \tag{A17}$$

which means that $s\left(x_2''\right) > s\left(x_2'\right)$, a contradiction since this means that $c_1\left(x_2'\right), c_2\left(x_2'\right), s_1\left(x_2'\right), s_2\left(x_2'\right)$ can not solve (A11) given $x_2 = x_2'$. Hence we conclude that $c\left(x_2''\right) < c\left(x_2'\right)$ and (symmetric argument) that $s\left(x_2''\right) > s\left(x_2'\right)$. But then, by Lemma 2 it follows that

$$g\left(c\left(x_2^\lambda\right), s\left(x_2^\lambda\right); \pi\right) > \min\left\{g\left(c\left(x_2'\right), s\left(x_2'\right); \pi\right), g\left(c\left(x_2''\right), s\left(x_2''\right); \pi\right)\right\} \geq 0, \tag{A18}$$

so $\left(c_1\left(x_2^\lambda\right), s_1\left(x_2^\lambda\right), c_2\left(x_2^\lambda\right), s_2\left(x_2^\lambda\right)\right)$ is not on the frontier of $Z(\pi)$, which means that $\left(x_1\left(x_2^\lambda\right), x_2^\lambda\right)$ is not on the frontier of $X(\pi)$, which contradicts the definition of $\left(x_1\left(x_2^\lambda\right), x_2^\lambda\right)$. ∎

Proof of Proposition 1. (only if) Suppose p^*, w^*, $x_1(w,p), x_2(w,p)$, (x_1^*, x_2^*) and $\{l_i^{c*}, l_i^{s*}\}_{i=1,2}$ satisfy conditions 1-4 in the definition of equilibrium given $\pi = \pi^*$ and let $c_i^* = \int l_i^{c*}(\theta)\, P(\theta, \pi)\, d\theta$ and $s_i^* = \int l_i^{s*}(\theta)\, d\theta$ for $i = 1, 2$.

Claim 1 $\Pi_i^* = p_i^* y^i\left(c_i^*, s_i^*\right) - \int w^*(\theta) \sum_{t=c,s} l_i^{t*}(\theta)\, d\theta = 0$ for $i = 1, 2$.

Proof. If $\Pi_i^* < 0$, then $l_i^{t\prime}(\theta) = 0$ is better than $\{l_i^{c*}, l_i^{s*}\}$ since it yields zero profits and if $\Pi_i^* > 0$, then $l_i^{t\prime}(\theta) = 2 l_i^{t*}(\theta)$ is better than $\{l_i^{c*}, l_i^{s*}\}$ since the associated profits are $2\Pi_i^*$. ∎

Claim 2 Let $\Theta^c = \{\theta | l_1^{c*}(\theta) + l_2^{c*}(\theta) > 0\}$ and $\Theta^s = \{\theta | l_1^{s*}(\theta) + l_2^{s*}(\theta) > 0\}$. Then there exists w_c, w_s such that $w^*(\theta) = w_c P(\theta, \pi)$ for almost all $\theta \in \Theta^c$ and $w^*(\theta) = w_s$ for almost all $\theta \in \Theta^s$.

Proof. For contradiction assume (without loss) that there is a constant κ and sets A, B where $A = \{\theta | w^* (\theta) > \kappa P (\theta, \pi)\}$ and $B = \{\theta | w^* (\theta) < \kappa P (\theta, \pi)\}$ and suppose that $\int_{\theta \in A} l_i^{c*} (\theta) \, d\theta > 0$ for some sector i. Consider an alternative labor demand by the firm where $l_i^{s\prime} = l_i^{s*}$ and $l_i^{c\prime} (\theta) = 0$ for all $\theta \in A$, $l_i^{c\prime} (\theta) = l_i^{c*} (\theta) + \beta$ for $\theta \in B$ and the demand for complex labor is unchanged for all other θ. We set β so that $\beta \int_{\theta \in B} P (\theta, \pi) \, d\theta = \int_{\theta \in A} P (\theta, \pi) \, l_i^{c*} (\theta) \, d\theta$, which means that effective factor inputs are the same as in the assumed equilibrium. The change in profits is thus the same as the change in wages, that is

$$\begin{aligned} \Pi_i' - \Pi_i^* &= -\int_{\theta \in B} \beta w^* (\theta) \, d\theta + \int_{\theta \in A} w^* (\theta) \, l_i^{c*} (\theta) \, d\theta = \\ &> -\kappa \int_{\theta \in B} \beta P (\theta, \pi) \, d\theta + \kappa \int_{\theta \in A} P (\theta, \pi) \, l_i^{c*} (\theta) \, d\theta = 0, \end{aligned} \tag{A19}$$

which contradicts the assumption that the original labor demand maximized profits given prices. Hence there must be some w_c such that $w^* (\theta) = w_c P (\theta, \pi)$ for almost all $\theta \in \Theta^c$. The second half follows by a symmetric argument. ■

Claim 3 $w^* (\theta) = \max (w_s, w_c P (\theta, \pi))$ for almost all $\theta \in [0, 1]$ and, ignoring deviations on sets of measure zero, $\Theta^s = [0, \theta^*]$ and $\Theta^c = [\theta^*, 1]$, where θ^* satisfies $\frac{w_s}{w_c} = P (\theta^*, \pi)$.

Proof. If there would be a set A with positive measure such that $w^* (\theta) < w_c P(\theta, \pi)$ for all $\theta \in A \subset [0, 1]$ or set $B \subset [0, 1]$ with positive measure such that $w^* (\theta) < w_s$ for all $\theta \in B$, then identical arguments as in the previous claim may be used to show that the presumed equilibrium demands can not be optimal given wages. Hence $w^* (\theta) \geq \max (w_s, w_c P (\theta, \pi))$ for almost all $\theta \in [0, 1]$. Moreover, if $w^* (\theta) > \max (w_s, w_c P (\theta, \pi))$ for θ on some set C, then $\sum_{i=1,2} \sum_{t=c,s} \int_{\theta \in C} l_i^t (\theta) \, d\theta = 0$ in order for firms to maximize profits, which contradicts condition 4 in the definition of equilibrium (factor market clearing), so $w^* (\theta) = \max (w_s, w_c P (\theta, \pi))$ for almost all $\theta \in [0, 1]$. Since $y^i (0, s) = y^i (c, 0) = 0$ and since $P (\theta, \pi)$ is strictly decreasing there must therefore be θ', θ'' such that $w_s > w_c P \left(\theta', \pi\right)$ and $w_s < w_c P \left(\theta'', \pi\right)$. Hence there is some $\theta^* \in (0, 1)$ such that $w_s = w_c P (\theta^*, \pi)$ and $w^* (\theta) = \max (w_s, w_c P (\theta, \pi))$ for almost all $\theta \in [0, 1]$. It follows by computations as in the previous claim that $\Theta^s \cap [\theta^*, 1]$ and $\Theta^c \cap [0, \theta^*]$ has measure zero. For example, if there is some set $D \subset [\theta^*, 1]$ with positive measure such that $l_i^{s*} (\theta) > 0$ for all $\theta \in D$, then $\int_{\theta \in D} l_i^{s*} (\theta) \, d\theta > 0$ and $\int_{\theta \in D} w^* (\theta) \, l_i^{s*} (\theta) \, d\theta > w_s \int_{\theta \in D} l_i^{s*} (\theta) \, d\theta$. Obviously, profits would be higher by letting $l_i^s (\theta) = l_i^{s*} (\theta) + \beta$ for $\theta \leq \theta^*$ and $l_i^s (\theta) = 0$ for $\theta > \theta^*$. ■

Claim 4 If x_1^*, x_2^* is part of an equilibrium with $p^* >> 0$, then x_1^*, x_2^* is on the frontier of $X(\pi)$.

Proof. If x_1^*, x_2^* is not on the frontier of $X(\pi)$ there is some $z' \in Z(\pi)$ such that $y^1(c_1', s_1') > x_1^*$ and $y^2(c_2', s_2') \geq x_2^*$. Total revenue in the economy increases and factor market clearing (condition 4 in Definition 1) implies that total costs are unchanged and equal to $\int w^*(\theta) f_\pi(\theta) d\theta$. Hence, at least one sector must make higher profits, which contradicts that the original labor demands were part of an equilibrium. ■

To complete the only if part we note that individual utility maximization implies that $p^*x > p^*x^*$ for all x such that $u(x_1, x_2) > u(x_1^*, x_2^*)$. By Claim 4, x^* is on the frontier of $X(\pi)$, so p^* must separate $X(\pi)$ and $\left\{x \in R_+^2 | u(x_1, x_2) > u(x_1^*, x_2^*)\right\}$ and the marginal conditions in (18) must hold. Using the structure on wages imposed by Claim 3 the profit maximization problem reduces to $\max_{c_i, s_i} p_i^* y^i(c_i, s_i) - w_c c_i - w_s s_i$, so

$$\begin{aligned} w_c &= p_1^* \frac{\partial y^1(c_1^*, s_1^*)}{\partial c_1} = p_2^* \frac{\partial y^2(c_2^*, s_2^*)}{\partial c_2} \text{ and} \\ w_s &= p_1^* \frac{\partial y^1(c_1^*, s_1^*)}{\partial s_1} = p_2^* \frac{\partial y^2(c_2^*, s_2^*)}{\partial s_2}. \end{aligned} \tag{A20}$$

Combining Claim 3 and factor market clearing we find that $c_1^* + c_2^* = \pi(1 - F_q(\theta^*))$, or $\theta^* = F_q^{-1}\left(\frac{\pi - c_1^* - c_2^*}{\pi}\right)$, which implies (again using Claim 3) that the wage scheme is of the form in (14). ■

Proof of Proposition 1. (if) Let (x_1^o, x_2^o) solve (12). $U = \left\{x \in R_+^2 | u(x_1, x_2) > u(x_1^o, x_2^o)\right\}$ is convex by assumption of preferences and $X(\pi)$ convex by 4. Moreover U and $X(\pi)$ are disjoint (since (x_1^o, x_2^o) solves (12)). It follows that there is a hyperplane separating U and $X(\pi)$. Let p^* be a normal to such an hyperplane and let $z^o = (c_1^o, s_1^o, c_2^o, s_2^o)$ be some factor inputs such that $x_i^o = y^i(c_i^o, s_i^o)$ for $i = 1, 2$ and let

$$w^*(\theta) = \begin{cases} p_1^* \frac{\partial y^1(c_1^o, s_1^o)}{\partial s_1} & \text{for } \theta \leq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ p_1^* P(\theta, \pi) \frac{\partial y^1(c_1^o, s_1^o)}{\partial c_1} & \text{or } \theta > F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases}. \tag{A21}$$

We note that since $\left\{x \in R_+^2 | px = p^*x\right\}$ is tangent to $X(\pi)$ at (x_1^o, x_2^o) it follows from (A12) that

$$\frac{p_1^*}{p_2^*} = \frac{\frac{\partial y(c_1^o, s_1^o)}{\partial c_2}}{\frac{\partial y(c_2^o, s_2^o)}{\partial c_1}} = \frac{\frac{\partial y(c_1^o, s_1^o)}{\partial s_2}}{\frac{\partial y(c_2^o, s_2^o)}{\partial c_1}}. \tag{A22}$$

Finally, let $l^o : \Theta \to R_+^4$ be any labor demand such that $c_i^o = \int_\theta P(\theta,\pi)\, l_i^c(\theta)\, d\theta$ and $s_i^o = \int_\theta l_i^s(\theta)\, d\theta$ and

$$
\begin{aligned}
l_1^c(\theta) + l_2^c(\theta) &= \begin{cases} 0 & \text{for } \theta < F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ f_\pi(\theta) & \text{for } \theta \geq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases} \\
l_1^s(\theta) + l_2^s(\theta) &= \begin{cases} f_\pi(\theta) & \text{for } \theta < F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ 0 & \text{for } \theta \geq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases},
\end{aligned}
\tag{A23}
$$

Existence of labor demands satisfying (A23) follows from the construction of the set of feasible factor inputs, $Z(\pi)$ and equilibrium condition 4 is satisfied by construction of (A23). Now, let $x(w,p)$ solve (1), so that the second equilibrium condition is satisfied, then aggregate demands can be found by integrating $x(w^*(\theta), p^*)$using the distribution $f_\pi(\theta)$. Since preferences are homothetic this implies that for the third condition (market clearing on goods) to be satisfied x_1^*, x_2^* must solve (17) given $p = p^*$ and $w = w^*$, which is the case since

$$
\begin{aligned}
\int w^*(\theta) f_\pi(\theta)\, d\theta &= \int_0^{\theta^*} w^*(\theta)\left(l_1^s(\theta) + l_2^s(\theta)\right) d\theta + \int_{\theta^*}^1 w^*(\theta)\left(l_1^c(\theta) + l_2^c(\theta)\right) d\theta = \\
&= \int_0^{\theta^*} \sum_{i=1,2} p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial s_i} l_i^s(\theta) + \int_{\theta^*}^1 \sum_{i=1,2} p_1^* P(\theta,\pi) \frac{\partial y^i(c_i^o, s_i^o)}{\partial c_i} l_i^c(\theta)\, d\theta = \\
&= \sum_{i=1,2} p_i^* y^i(c_i^o, s_i^o) = p_1^* x_1^* + p_2^* x_2^*,
\end{aligned}
\tag{A24}
$$

after use of constant returns to scale. Thus, since p^* is tangent to $X(\pi)$, the aggregate bundle (x_1^*, x_2^*) must also solve $\max_{x_1,x_2} u(x_1,x_2)$ subj. to. $p^* x \leq p^* x^*$ and by the calculation above this will be the case. Left to verify is that $l^o : \Theta \to R_+^4$ is consistent with the first equilibrium condition, profit maximization for the firms. To see this, consider any alternative labor demand for firm i, which generates profits $\Pi_i' = p_i^* y^i(c_i', s_i') - \int_\theta w^*(\theta)\left(l_i^{s\prime}(\theta) + l_i^{c\prime}(\theta)\right) d\theta$. From (A21) and (A22) it follows that $w^*(\theta) = \max\left\{p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial s_i}, p_1^* P(\theta,\pi) \frac{\partial y^i(c_i^o, s_i^o)}{\partial c_i}\right\}$, so

$$
\begin{aligned}
\int_\theta w^*(\theta) l_i^{s\prime}(\theta)\, d\theta &\geq \int_\theta p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial s_i} l_i^{s\prime}(\theta)\, d\theta = p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial s_i} s_i' \\
\int_\theta w^*(\theta) l_i^{c\prime}(\theta)\, d\theta &\geq \int_\theta p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial c_i} P(\theta,\pi)\, l_i^{s\prime}(\theta)\, d\theta = p_i^* \frac{\partial y^i(c_i^o, s_i^o)}{\partial c_i} c_i'
\end{aligned}
\tag{A25}
$$

and we have that

$$
\Pi_i' \leq p_i^* \left(y^i(c_i', s_i') - \frac{\partial y^i(c_i^o, s_i^o)}{\partial s_i} s_i' - \frac{\partial y^i(c_i^o, s_i^o)}{\partial c_i} c_i' \right) \leq \tag{A26}
$$

$$\leq \; p_i^* \left(y^i \left(c_i^o, s_i^o\right) + \frac{\partial y^i \left(c_i^o, s_i^o\right)}{\partial s_i} \left(c_i' - c_i^o\right) + \frac{\partial y^i \left(c_i^o, s_i^o\right)}{\partial c_i} \left(c_i' - c_i^o\right) \right)$$
$$-p_i^* \left(\frac{\partial y^i \left(c_i^o, s_i^o\right)}{\partial s_i} s_i' - \frac{\partial y^i \left(c_i^o, s_i^o\right)}{\partial c_i} c_i' \right) \; = \; 0.$$

A routine calculation verifies that profits are zero if firms hire in accordance to l^o, so this means that all equilibrium conditions 1-4 are satisfied. ■

Proof of Proposition 2. (Continuity) Since the arguments are standard we will only give a sketch of the proof. By Proposition 1 we know that

$$x(\pi) = (x_1(\pi), x_2(\pi)) = \arg \max_{x \in X(\pi)} u(x_1, x_2), \tag{A27}$$

Observe that the constraint may be viewed as a correspondence $X : [0,1] \rightrightarrows R_+^2$ and it is trivial to verify that the constraint correspondence is compact-valued and continuous (i.e. uhc and lhc). It then follows directly from the theorem of the maximum that the set of maximizers is upper-hemi continuous. Since there is a unique maximizer for each π, this implies continuity of $x(\pi)$.. Next one inspects (A11) and observe that the associated factor inputs $z(\pi)$ solves

$$\begin{aligned} z(\pi) \;&=\; (c_1(\pi), s_1(\pi), c_2(\pi), s_2(\pi)) \;&=\;& \arg\max_z y^1(c_1, s_1) \\ &\text{s.t } y^2(c_2, s_2) \;&\geq\;& x_2(\pi), .g(c_1 + c_2, s_1 + s_2; \pi) \geq 0 \end{aligned} \tag{A28}$$

Let the constraint correspondence for this problem be denoted as $\Gamma : [0,1] \rightrightarrows R_+^4$ and note that

$$\Gamma(\pi) = \left\{ z \in Z(\pi) \,|\, y^2(c_2, s_2) \geq x_2(\pi) \right\}, \tag{A29}$$

which obviously is closed, bounded and continuous. Hence, the theorem of the maximum applies again and since y^i is strictly quasi concave and the constraint set is convex, $z(\pi)$ is uniquely defined. Thus, also the factor inputs are continuous functions of π and direct inspection of the expression for equilibrium wages reveals that $w(\theta, \pi)$ is continuous in π for each $\theta \in [0,1]$. Indeed, it is not hard to verify that for each $\pi \in (0,1)$ and $\delta > 0$ there exists $\varepsilon > 0$ such that $\|w(\theta,\pi) - w(\theta,\pi')\| < \delta$ for all $\theta \in [0,1]$ and since compositions of continuous functions are continuous and $p(\pi)$ is continuous (otherwise $x(\pi)$ can't be continuous) continuity of B follows. That $B(1) = B(0) = 0$ is easy to see directly from (19). We leave it to the reader to check that $B(\pi) > 0$ for all $\pi \in (0,1)$, which follows from a direct computation using the monotone likelihood ratio. ■

Proof of Proposition 3. Opening up a country for trade has no effect on the profit maximization problem for an individual firm or the conditions for factor market clearing. Hence, all arguments in the proof of Proposition 1 that were used in the characterization of equilibrium wages apply also with trade, so wages must be consistent with (24). To see that x_w^* must be on the frontier of $X_w(\pi)$, suppose that this is not the case. Then there exists alternative factor inputs so that $x_{iw}^* \le \lambda_h y^i(c'_{ih}, s'_{ih}) + \lambda_f y^i\left(c'_{if}, s'_{if}\right)$for both goods and with strict inequality for one good. Hence

$$\begin{aligned} p_1^* x_{1w}^* + p_2^* x_{2w}^* &< \lambda_h p_1^* y^1\left(c'_{1h}, s'_{1h}\right) + \lambda_f p_1^* y^1\left(c'_{1f}, s'_{1f}\right) \\ &\quad + \lambda_h p_2^* y^2\left(c'_{2h}, s'_{2h}\right) + \lambda_f p_1^* y^2\left(c'_{2f}, s'_{2f}\right) \end{aligned} \tag{A30}$$

Moreover, for the alternative inputs to be feasible it must be that $\sum_i \sum_t l_{ij}^{t\prime}(\theta) \le f_\pi(\theta) = \sum_i \sum_t l_{ij}^{t*}(\theta)$ for each country and every θ. Let

$$W_{ij}^* = \int_\theta w^*(\theta)\left(l_i^{c*}(\theta) + l_i^{c*}(\theta)\right) d\theta \text{ and } W'_{ij} = \int_\theta w'(\theta)\left(l_i^{c\prime}(\theta) + l_i^{c\prime}(\theta)\right) d\theta \tag{A31}$$

be the wage costs in the assumed equilibrium and for the alternative factor inputs respectively. Clearly $W'_{1j} + W'_{2j} \le W_{1j}^* + W_{2j}^*$ for $j = h, f$, which combined with (A30) yields

$$\sum_{j=h,f} \lambda_j \left(p_1^* x_{1j}^* + p_2^* x_{2j}^* - W_{1j}^* + W_{2j}^*\right) < \sum_{j=h,f} \lambda_j \left(p_1^* y^1\left(c'_{1j}, s'_{1j}\right) + p_2^* y^2\left(c'_{2j}, s'_{2j}\right) - W'_{1j} + W'_{2j}\right), \tag{A32}$$

which means that there must be at least one sector in at least one country where profits are not maximized. Thus x_w^* must be on the frontier of $X_w(\pi)$ and since utility maximization implies that all bundles better than x_w^* must be strictly more expensive given world prices this completes the proof of the "only if" part. The "if" part follows step by step the autarky case. ■

Proof of Proposition 4 (sketch). Uniqueness of x_w^* is immediate from Proposition 3. For contradiction suppose that (x_h^*, x_f^*) and (x_h^{**}, x_f^{**}) are distinct country specific equilibrium outputs that generate x_w^*. By strict convexity of $X(\pi_h)$ and $X(\pi_f)$ it is then possible to find $x'_w >> x_w^*$ that is in $X_w(\pi)$, which contradicts that x_w^* is on the frontier of $X_w(\pi)$. Hence, (x_h^*, x_f^*) are unique and uniqueness of $(c_{1j}^*, c_{2j}^*, s_{1j}^*, s_{2j}^*)$ can be established exactly as in the model without trade. ■

Proof of Proposition 5 (sketch): Let $b^i(w) = \min_{c,s} w_c c + w_s s$ subject to $f^i(c,s) \le 1$. Mathematically this is the same object as the unit cost function in textbook trade theory. Let w^h and w^f denote the effective factor prices in each country. If both goods are produced in both countries it follows that $b^i(w^h) = b^i(w^f) = p_i$ for each industry i. It is easy to check that the single-crossing condition on the isoquants (assumption **A1)** implies that the level curves associated with $b^1(w) = p_1$

are everywhere steeper than those for $b^2(w) = p_2$. Since goods prices are the same in both countries this proves that effective factor prices must be the same in both countries. ■

Proof of Proposition 6. For contradiction, suppose that $\pi_h < \pi_f$ and that country h does not import the skill intensive good. Suppose first that the equilibrium is such that both goods are produced by both countries. Since the two goods are consumed at the same ratio in each country we than have $x^*_{1h}/x^*_{2h} \geq x^*_{1f}/x^*_{2f}$ and $(w^*_{ch}, w^*_{sh}) = (w^*_{cf}, w^*_{sf})$. Inspection of the reduced form profit maximization problem then shows that the factor ratio is the same in both industries. For brevity let $\theta^*_j = \theta(c^*_{1j} + c^*_{2j}, \pi_j)$ and observe that (15), together with equal factor prices imply that $P(\theta^*_h, \pi_h) = P(\theta^*_f, \pi_f)$, which (since $\pi_h < \pi_f$ and P strictly increasing in both arguments) implies that $\theta^*_h > \theta^*_f$, so,

$$c^*_{1h} + c^*_{2h} = \pi_h(1 - F_q(\theta^*_h) < \pi_f(1 - F_q(\theta^*_f) = c^*_{1f} + c^*_{2f}. \tag{A33}$$

Moreover, $F_q(\theta) < F_u(\theta)$ for all $\theta < 1$ since if for some $\theta' < 1$ we have that $F_q(\theta') = F_u(\theta')$, then the monotone likelihood ratio assumption can not hold, so

$$\begin{aligned} s^*_{1h} + s^*_{2h} &= \pi_h F_q(\theta^*_h) + (1 - \pi_h) F_q(\theta^*_h) > \pi_f F_q(\theta^*_h) + (1 - \pi_f) F_q(\theta^*_h) > \\ &> \pi_f F_q(\theta^*_f) + (1 - \pi_f) F_q(\theta^*_f) = s^*_{1f} + s^*_{2f} \end{aligned} \tag{A34}$$

From constant returns it follows that

$$\frac{s^*_{1h}}{s^*_{1f}} = \frac{c^*_{1h}}{c^*_{1f}} = \frac{x^*_{1h}}{x^*_{1f}} \geq \frac{x^*_{2h}}{x^*_{2f}} = \frac{c^*_{2h}}{c^*_{2f}} = \frac{s^*_{2h}}{s^*_{2f}} \tag{A35}$$

which implies that

$$c^*_{1h} + c^*_{2h} = c^*_{1f}\frac{x^*_{1h}}{x^*_{1f}} + c^*_{2f}\frac{x^*_{2h}}{x^*_{2f}} < c^*_{1f} + c^*_{2f} \Rightarrow 0 > c^*_{1f}\left(\frac{x^*_{1h}}{x^*_{1f}} - 1\right) + c^*_{2f}\left(\frac{x^*_{2h}}{x^*_{2f}} - 1\right) \tag{A36}$$

$$\begin{aligned} s^*_{1h} + s^*_{2h} &= s^*_{1f}\frac{x^*_{1h}}{x^*_{1f}} + s^*_{2f}\frac{x^*_{2h}}{x^*_{2f}} > s^*_{1f} + s^*_{2f} \Rightarrow \\ 0 &< s^*_{1f}\left(\frac{x^*_{1h}}{x^*_{1f}} - 1\right) + s^*_{2f}\left(\frac{x^*_{2h}}{x^*_{2f}} - 1\right) < s^*_{1f}\left(\frac{x^*_{1h}}{x^*_{1f}} - 1\right) - s^*_{2f}\frac{c^*_{1f}}{c^*_{2f}}\left(\frac{x^*_{1h}}{x^*_{1f}} - 1\right) \end{aligned} \tag{A37}$$

All factor inputs are positive, so to satisfy (A37) one coefficient must be strictly positive and combining with (A35) we have that $\left(\frac{x^*_{1h}}{x^*_{1f}} - 1\right) > 0$, hence

$$0 < s^*_{1f} - s^*_{2f}\frac{c^*_{1f}}{c^*_{2f}} \Leftrightarrow s^*_{2f}\frac{c^*_{1f}}{c^*_{2f}} < s^*_{1f} \Rightarrow \frac{c^*_{1f}}{s^*_{1f}} < \frac{c^*_{2f}}{s^*_{2f}}, \tag{A38}$$

which is a contradiction since the factor intensity assumption is violated.

There are a few possibilities left to rule out for cases when factor price equalization doesn't hold. First it may be that country h only produces good 1 and country 2 only produces good 2. Here (A33) and (A34) gives an immediate violation of the factor intensity assumption. Next, country h may only produce good 1 while the production is diversified in the other country. The first 3 equalities in (A35) still applies, so

$$\begin{aligned} c_{1h}^* &= c_{1f}^* \frac{x_{1h}^*}{x_{1f}^*} < c_{1f}^* + c_{2f}^* \Rightarrow c_{1f}^* \left(\frac{x_{1h}^*}{x_{1f}^*} - 1 \right) < c_{2f}^* \\ s_{1h}^* &= s_{1f}^* \frac{x_{1h}^*}{x_{1f}^*} > s_{1f}^* + s_{2f}^* \Rightarrow s_{1f}^* \left(\frac{x_{1h}^*}{x_{1f}^*} - 1 \right) > s_{2f}^* \end{aligned} \tag{A39}$$

All inputs in country f are strictly positive, so $\left(\frac{x_{1h}^*}{x_{1f}^*} - 1 \right) > 0$, which leads to a violation of the factor intensity assumption. The final possibility is when country h is diversified and country 2 only produces good 2 and the argument follows the same line of reasoning as the previous argument. ■

Proof of Lemma 4. For brevity we drop the country index j and let $p_1 = p(\pi), p_1' = p(\pi')$, $p_2 = p_2' = 1$, $c = c_j(\pi)$, $c' = c_j(\pi')$, $c_i = c_{ij}(\pi)$ for $i = 1, 2$, $c_i' = c_{ij}(\pi')$ and so on. Since (c_i, s_i) is a solution to the profit maximization problem given prices (p_i, w_c, w_s) and since (c_i', s_i') is a solution to the profit maximization problem given prices (p_i', w_c', w_s') it follows that

$$\begin{aligned} p_i y^i(c_i, s_i) - w_c c_i - w_s s_i &\geq p_i y^i(c_i', s_i') - w_c c_i' - w_s s_i' \\ p_i' y^i(c_i', s_i') - w_c' c_i' - w_s' s_i' &\geq p_i' y^i(c_i, s_i) - w_c' c_i - w_s' s_i \end{aligned} \tag{A40}$$

for $i = 1, 2$ and using strict quasi-concavity of y^i is easy to show that if $\frac{c_i}{s_i} \neq \frac{c_i'}{s_i'}$, then

$$\begin{aligned} p_i y^i(c_i, s_i) - w_c c_i - w_s s_i &> p_i y^i(c_i', s_i') - w_c c_i' - w_s s_i' \text{ and} \\ p_i' y^i(c_i', s_i') - w_c' c_i' - w_s' s_i' &> p_i' y^i(c_i, s_i) - w_c' c_i - w_s' s_i \end{aligned} \tag{A41}$$

Now let x_i and x_i' denote the equilibrium outputs $x_{ij}(\pi)$ (equal to $y^i(c_i, s_i)$), add and rearrange to get

$$\sum_i (p_i - p_i')(x_i - x_i') \geq (w_c - w_c')(c - c') + (w_s - w_s')(s - s') \tag{A42}$$

and the proof is then complete by observing that $p_2 = p_2' = 1$ by choice of unit of account. ■

Proof of Proposition 7. If $x_{ij}(\pi), x_{ij}(\pi') > 0$ for all i, j Proposition 5 implies that for both i we have that

$$\frac{\partial y^2(r_{2h}(\pi), 1)}{\partial s} = w_s(\pi) = \frac{\partial y^2(r_{2b}(\pi), 1)}{\partial s} \tag{A43}$$

$$\frac{\partial y^2\left(r_{2h}\left(\pi\right),1\right)}{\partial c} = w_c\left(\pi\right) = \frac{\partial y^2\left(r_{2b}\left(\pi\right),1\right)}{\partial c}.$$

Moreover, the equilibrium wage schemes are also such that the technical rate of substitution of factors are equalized across sectors, i.e.,

$$\frac{\frac{\partial y^1(r_{1j}(\pi),1)}{\partial c}}{\frac{\partial y^1(r_{1j}(\pi),1)}{\partial s}} = \frac{\frac{\partial y^2(r_{2j}(\pi),1)}{\partial c}}{\frac{\partial y^2(r_{2j}(\pi),1)}{\partial s}}. \tag{A44}$$

The same equalities hold also for π'. This implies that:

1) if $w_c\left(\pi\right) \leq w_c\left(\pi'\right)$, then $r_{2j}\left(\pi\right) \geq r_{2j}\left(\pi'\right)$ for $j = h, f \Rightarrow w_s\left(\pi\right) \geq w_s\left(\pi'\right)$ (from (A43))

2) similarly, if $w_s\left(\pi\right) \geq w_s\left(\pi'\right)$, then $r_{2j}\left(\pi\right) \geq r_{2j}\left(\pi'\right)$ for $j = h, f \Rightarrow w_c\left(\pi\right) \leq w_c\left(\pi'\right)$ (from (A43))

3) if $r_{2j}\left(\pi\right) \geq r_{2j}\left(\pi'\right)$ then $r_{1j}\left(\pi\right) \geq r_{1j}\left(\pi'\right)$ (from (A44))

Hence, a failure of the result could only happen if $w_c\left(\pi\right) \leq w_c\left(\pi'\right)$ and $w_s\left(\pi\right) \geq w_s\left(\pi'\right)$ and $r_{ij}\left(\pi\right) \geq r_{ij}\left(\pi'\right)$ for all i, j if this would be the case. But from the characterization of the equilibrium wage scheme we have that

$$P\left(\theta_j\left(\pi\right),\pi\right)\frac{\partial y^i\left(r_{ij}\left(\pi\right),1\right)}{\partial c} = \frac{\partial y^i\left(r_{ij}\left(\pi\right),1\right)}{\partial s} \tag{A45}$$

for all i, j and similarly for π', so

$$P\left(\theta_j\left(\pi\right),\pi\right) = \frac{\frac{\partial y^i(r_{ij}(\pi),1)}{\partial s}}{\frac{\partial y^i(r_{ij}(\pi),1)}{\partial c}} \geq \frac{\frac{\partial y^i(r_{ij}(\pi'),1)}{\partial s}}{\frac{\partial y^i(r_{ij}(\pi'),1)}{\partial c}} = P\left(\theta_j\left(\pi'\right),\pi'\right). \tag{A46}$$

Since P is strictly increasing in both arguments this means that $\theta_h\left(\pi\right) > \theta_h\left(\pi'\right)$ and $\theta_f\left(\pi\right) \geq \theta_f\left(\pi'\right)$ and this means that we can sign the changes in total efficient inputs of the two factors as

$$\begin{aligned} c_j\left(\pi\right) &= \pi_j(1 - F_q(\theta_j\left(\pi\right))) \leq \pi_j'(1 - F_q(\theta_j(\pi'))) = c_j\left(\pi'\right), \\ s_j\left(\pi\right) &= \pi_j F_{\pi_j}(\theta_j\left(\pi\right)) \geq \pi_j' F_{\pi_j'}(\theta_j\left(\pi'\right)) = s_j\left(\pi'\right) \end{aligned} \tag{A47}$$

with strict inequalities for $j = h$ and the second inequality follows since $F_\pi\left(\theta\right) = \pi F_q\left(\theta\right) + \left(1-\pi\right)F_u\left(\theta\right)$ is decreasing in π by the monotone likelihood ratio property and increasing in θ since it is a cumulative distribution function (same calculation as in the proof of Proposition 6). Adding the inequalities in (A47) we get that

$$\begin{aligned} c_w\left(\pi\right) &\equiv c_h\left(\pi\right) + c_f\left(\pi\right) < c_h\left(\pi'\right) + c_f\left(\pi'\right) \equiv c_w\left(\pi'\right) \\ s_w\left(\pi\right) &\equiv s_h\left(\pi\right) + s_f\left(\pi\right) > s_h\left(\pi'\right) + s_f\left(\pi'\right) \equiv s_w\left(\pi'\right) \end{aligned} \tag{A48}$$

Hence, in spite of all factor ratios being higher in the equilibrium with the lower π it must be that the aggregate use of complex labor is lower in the equilibrium with lower π and that the aggregate use of simple labor is higher. We now let $c_{iw}(\pi) = c_{ih}(\pi) + c_{if}(\pi)$ denote the sector specific use of complex labor in the world and let $s_{iw}(\pi)$ be defined in the same way and note that $c_{iw}(\pi)/s_i(\pi)$ $= r_{iw}(\pi) = r_{ih}(\pi) = r_{if}(\pi)$ and similarly for π'. We have that

$$s_w(\pi') = \frac{c_{1w}(\pi')}{r_{1w}(\pi')} + \frac{c_{2w}(\pi')}{r_{2w}(\pi')} = c_{1w}(\pi')\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi')}{r_{2w}(\pi')}, \tag{A49}$$

and, since $r_{iw}(\pi) \geq r_{iw}(\pi')$ for $i = 1, 2$,

$$\begin{aligned} s_w(\pi) &= s_{1w}(\pi) + s_{2w}(\pi) = \frac{c_{1w}(\pi)}{r_{1w}(\pi)} + \frac{c_{2w}(\pi)}{r_{2w}(\pi)} \leq \\ &\leq \frac{c_{1w}(\pi)}{r_{1w}(\pi')} + \frac{c_{2w}(\pi)}{r_{2w}(\pi')} = \frac{c_{1w}(\pi)}{r_{1w}(\pi')} + \frac{(c_w(\pi) - c_{1w}(\pi))}{r_{2w}(\pi')} = \\ &= c_{1w}(\pi)\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi)}{r_{2w}(\pi')}. \end{aligned} \tag{A50}$$

Hence by combining (A49) and (A50) we get that

$$0 < s_w(\pi) - s_w(\pi') \leq (c_{1w}(\pi) - c_{1w}(\pi'))\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi) - c_w(\pi')}{r_{2w}(\pi')}. \tag{A51}$$

Since $c_w(\pi) < c_w(\pi')$ and, by the factor intensity assumption, $r_{1w}(\pi') > r_{2w}(\pi')$ it follows that a necessary condition for (A51) to hold is that $c_{1w}(\pi) < c_{1w}(\pi')$. Similar algebra also reveals that

$$\begin{aligned} s_w(\pi') &= \frac{c_{1w}(\pi')}{r_{1w}(\pi')} + \frac{c_{2w}(\pi')}{r_{2w}(\pi')} = c_{2w}(\pi')\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi')}{r_{1w}(\pi')} \\ s_w(\pi) &= \frac{c_{1w}(\pi)}{r_{1w}(\pi)} + \frac{c_{2w}(\pi)}{r_{2w}(\pi)} \leq c_{2w}(\pi)\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi)}{r_{1w}(\pi')}, \end{aligned} \tag{A52}$$

implying that

$$0 < s_w(\pi) - s_w(\pi') \leq (c_{2w}(\pi) - c_{2w}(\pi'))\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi) - c_w(\pi')}{r_{1w}(\pi')}, \tag{A53}$$

so $c_{2w}(\pi) > c_{2w}(\pi')$ is also necessary under the hypothesis of the claim. In the same spirit we have that

$$\begin{aligned} c_w(\pi') &= (r_{1w}(\pi') - r_{2w}(\pi'))\, s_{1w}(\pi') + r_2(\pi')\, s_w(\pi') \\ c_w(\pi) &\geq (r_{1w}(\pi') - r_{2w}(\pi'))\, s_{1w}(\pi) + r_2(\pi')\, s_w(\pi), \end{aligned} \tag{A54}$$

so that

$$0 > c_w(\pi) - c_w(\pi') \geq (r_{1w}(\pi') - r_{2w}(\pi'))\,(s_{1w}(\pi) - s_{1w}(\pi')) + r_2(\pi')\,(s_w(\pi) - s_w(\pi')), \tag{A55}$$

which implies that $s_{1w}(\pi) < s_{1w}(\pi')$. Finally, we proceed in the same way to show that

$$\begin{aligned} c_w(\pi') &= (r_{2w}(\pi') - r_{1w}(\pi'))\, s_{2w}(\pi') + r_1(\pi')\, s_w(\pi') \\ c_w(\pi) &\geq (r_{2w}(\pi') - r_{1w}(\pi'))\, s_{2w}(\pi) + r_1(\pi')\, s_w(\pi), \end{aligned} \tag{A56}$$

which can be combined as

$$0 > c_w(\pi) - c_w(\pi') \geq (r_{2w}(\pi') - r_{1w}(\pi'))\,(s_{2w}(\pi) - s_{2w}(\pi')) + r_1(\pi')\,(s_w(\pi) - s_w(\pi')) \tag{A57}$$

implying that $s_{2w}(\pi) > s_{2w}(\pi')$. Hence, since both inputs in industry 1 (2) are higher (lower) in the equilibrium with investments according to π' than with π it follows that

$$\begin{aligned} x_{1w}(\pi) &= y^1(c_{1w}(\pi), s_{1w}(\pi)) < y^1(c_{1w}(\pi'), s_{1w}(\pi')) = x_{1w}(\pi') \\ x_{2w}(\pi) &= y^2(c_{2w}(\pi), s_{2w}(\pi)) > y^2(c_{2w}(\pi'), s_{2w}(\pi')) = x_{2w}(\pi'). \end{aligned} \tag{A58}$$

To finish up the proof of the result we note that $x_{1w}(\pi) < x_{1w}(\pi')$ and $x_{2w}(\pi) > x_{2w}(\pi')$ implies that $\frac{x_{1w}(\pi)}{x_{2w}(\pi)} < \frac{x_{1w}(\pi')}{x_{2w}(\pi')}$, which in turn implies that $p(\pi) > p(\pi')$ from optimal consumer behavior. Moreover, with factor price equalization Lemma 7 implies that

$$\begin{aligned} 0 > \underbrace{(p(\pi) - p(\pi'))}_{>0}\underbrace{(x_{1w}(\pi) - x_{1w}(\pi'))}_{<0} \geq\; & (w_c(\pi) - w_c(\pi'))\,\underbrace{(c_w(\pi) - c_w(\pi'))}_{<0} \\ & + (w_s(\pi) - w_s(\pi'))\,\underbrace{(s_w(\pi) - s_w(\pi'))}_{>0}. \end{aligned} \tag{A59}$$

But, $w_c(\pi) \leq w_c(\pi')$ and $w_s(\pi) \geq w_s(\pi')$ under the assumption that the proposition fails, implying that the right hand side is positive, a contradiction. ■

Proof of Proposition 8. We only prove the result for the case with $\alpha \leq \eta$. The case with $\alpha > \eta$ proceeds along the same lines, but the calculations are different. We first consider the case with a uniform distribution and then use a linear approximation and concavity to generalize the result to arbitrary concave functions. In general, since $G(B(0)) > 0$ by the assumption that $\underline{k} < 0$ and since $G(B(\pi)) > G(B(1)) = G(B(0))$ for all $\pi \in (0, 1)$ it follows that there must be some equilibrium $\pi^* > 0$. In case $\pi^* = 1$ uniqueness follows trivially since then $G(B(\pi)) > G(B(1)) \geq 1$ for all $\pi < 1$, implying that there is no other equilibrium. Hence, consider $\pi^* < 1$ in any equilibrium. A sufficient condition for there to be a unique equilibrium is that $\frac{d}{d\pi}G(B(\pi^*)) < 1$ in any equilibrium π^* and we prove the proposition by a contradiction argument that shows that this must be the case. Define

$$b(\pi) \equiv \left(\frac{\pi}{\eta - k\pi}\right)^{\alpha} \left(\frac{\alpha}{k\pi + (1-\eta)} - 1\right). \tag{A60}$$

With k distributed uniformly on $[\underline{k}, \overline{k}]$ we then have that $G(B(\pi)) = Qb(\pi) + L$ for some positive constants Q and L and any $B(\pi) \in [\underline{k}, \overline{k}]$.[11] By a direct calculation we have that

$$\begin{aligned} b'(\pi) &= \alpha\left(\frac{\pi}{\eta - k\pi}\right)^{\alpha-1} \left(\frac{\alpha}{k\pi + (1-\eta)} - 1\right) \frac{\eta}{(\eta - k\pi)^2} - \left(\frac{\pi}{\eta - k\pi}\right)^{\alpha} \frac{k\alpha}{(k\pi + (1-\eta))^2} \\ &= b(\pi)\frac{\alpha\eta}{\pi(\eta - k\pi)} - b(\pi)\frac{k\alpha}{(k\pi + (1-\eta))^2}\frac{k\pi + (1-\eta)}{\alpha - k\pi - (1-\eta)} \\ &= b(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta - k\pi)} - \frac{k}{(k\pi + (1-\eta))(\alpha - k\pi - (1-\eta))}\right). \end{aligned} \tag{A61}$$

But $\frac{d}{d\pi}G(B(\pi)) \geq 1$ holds if and only if $Qb'(\pi) \geq 1$, which since π is assumed to be an equilibrium point (satisfying $\pi = G(B(\pi)) = Kb(\pi) + L$) implies,

$$\begin{aligned} Qb'(\pi) &= Qb(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta - k\pi)} - \frac{k}{(k\pi + (1-\eta))(\alpha - k\pi - (1-\eta))}\right) \\ &= \alpha(\pi - L)\left(\frac{\eta}{\pi(\eta - k\pi)} - \frac{k}{(k\pi + (1-\eta))(\alpha - k\pi - (1-\eta))}\right) \geq 1 \end{aligned} \tag{A62}$$

But,

$$\begin{aligned} & \alpha(\pi - L)\left(\frac{\eta}{\pi(\eta - k\pi)} - \frac{k}{(k\pi + (1-\eta))(\alpha - k\pi - (1-\eta))}\right) \\ \leq\ & \alpha\pi\left(\frac{\eta}{\pi(\eta - k\pi)} - \frac{k}{(k\pi + (1-\eta))(\alpha - k\pi - (1-\eta))}\right) \Big/ \begin{matrix} k\pi + 1 - \eta \leq \alpha \\ \Rightarrow -\frac{1}{k\pi+1-\eta} \leq \frac{1}{\alpha} \end{matrix} \Big/ \\ \leq\ & \frac{\alpha\eta}{(\eta - k\pi)} - \frac{k}{(\alpha - k\pi - (1-\eta))} \Big/ \frac{k}{(\alpha - k\pi - (1-\eta))} \text{ increasing in } \pi \Big/ \\ \leq\ & \frac{\alpha\eta}{(\eta - k\pi)} - \frac{k}{(\alpha + \eta - 1)} \Big/ \begin{matrix} \eta \geq \alpha \Rightarrow k = 2\eta - 1 \geq \alpha + \eta - 1 \\ \Rightarrow \frac{k}{(\alpha+\eta-1)} \geq 1 \end{matrix} \Big/ \leq \frac{\alpha\eta}{(\eta - k\pi)} - 1 \end{aligned} \tag{A63}$$

and $B(\pi) = 0$ for all π such that $\pi > \frac{\alpha+\eta-1}{2\eta-1}$, so $\pi \leq \frac{\alpha+\eta-1}{2\eta-1} = \frac{a+\eta-1}{k}$ (and $\alpha \geq 1-\eta$ in the parameter range under consideration), implying that

$$\frac{\alpha\eta}{(\eta - k\pi)} \leq \frac{\alpha\eta}{(\eta - a - \eta + 1)} = \frac{\alpha\eta}{1-\alpha} \leq \frac{\eta^2}{1-\eta} < 2, \tag{A64}$$

since $\eta < 1$. Hence, $Qb'(\pi) < 1$, a contradiction. This ends the proof for the case with a uniform distribution. For a general concave distribution, interpret $G'(\underline{c})$ as the right derivative of G at $\underline{c}$

[11] Where in terms of the parametrs of the model, $Q = \frac{1}{\overline{k}-\underline{k}}kp^{\alpha}$ and $L = -\frac{\underline{k}}{\overline{k}-\underline{k}} > 0$ given that $\underline{c} < 0$.

and observe that, by concavity

$$\begin{aligned} \frac{d}{d\pi} G(B(\pi)) &= G'(B(\pi)) B'(\pi) \leq G'(\underline{c}) B'(\pi) \\ G(B(\pi)) &\leq G(\underline{c}) + G'(\underline{c})(B(\pi) - \underline{c}) = G'(\underline{c})(B(\pi) - \underline{c}) \end{aligned} \tag{A65}$$

Hence, if $\frac{d}{d\pi} G(B(\pi)) \geq 1$ at an equilibrium value for π it is necessary that

$$\begin{aligned} G'(\underline{c}) B'(\pi) &= G'(\underline{c}) k \eta^{\alpha} b'(\pi) \geq 0 \text{ and} \\ \pi &= G(B(\pi)) \leq G'(\underline{c})(B(\pi) - \underline{c}) \end{aligned} \tag{A66}$$

At this point let $Q = G'(\underline{c}) k \eta^{\alpha}$ and $L = -G'(\underline{c})\underline{c} > 0$ and proceed as with a uniform distribution. ■

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