Journal of Public Economics 87 (2003) 567–594
JOURNAL OF PUBLIC ECONOMICS
www.elsevier.com/locate/econbase

Affirmative action in a competitive economy

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Received 16 March 2001; accepted 13 April 2001

Abstract

We consider a model of endogenous human capital formation with competitively determined wages. Discrimination is sustainable in equilibrium in the presence of two distinguishable, but *ex ante identical* groups of workers. An affirmative action policy consisting of a quota may 'fail' in the sense that there still may be equilibria where groups are treated differently. However, the incentives to invest for agents in the discriminated group are improved by affirmative action if the initial equilibrium is the most discriminatory equilibrium in the model without the policy. The welfare effects are ambiguous. It is possible that the policy makes the intended beneficiaries worse off: even if the starting point is the most discriminatory equilibrium the expected payoff may decrease for all agents in the target group.

Keywords: Affirmative action; Statistical discrimination; General equilibrium; Asymmetric information; Human capital

JEL classification: D58; D62; D82; J71; J78

1. Introduction

Coate and Loury (1993) provided a new reason for caution with preferential policies above and beyond the usual efficiency arguments against quotas. They study a model of statistical discrimination in job assignments where identifiable groups are identical ex ante and discrimination is sustained as a self-confirming prophecy due to feedback effects between expected job assignments and incentives

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0047-2727/02/$ – see front matter
doi:10.1016/S0047-2727(01)00121-9

to invest in human capital. Under some circumstances affirmative action removes all equilibria with discrimination, but under equally plausible circumstances there are still equilibria where groups behave differently and employers rationally perceive members of one of the groups to be less capable. Affirmative action may even reduce the incentives to invest in human capital for the discriminated group, increasing disparity in investments. The policy may thus encourage employers to *patronize* the disadvantaged group instead of solving the underlying problem.

An important weakness with Coate and Loury's argument is that their model is not fully competitive. Employers assign a random sample of workers to jobs on the basis of observable characteristics and equilibrium investment behavior. Job assignments are chosen in order to maximize profits given some exogenously fixed wages that do not adjust to changes in market conditions, such as changes in investment behavior or the introduction of the affirmative action policy. This may seem like an innocuous simplification. After all, competitive firms take equilibrium wages as exogenously given, so it may appear that fixing wages exogenously may not be much of an issue. The problem with this is that a *policy intervention* changes the profitability on hiring different groups of workers. Since discrimination is best viewed as an economy-wide problem one would therefore think that an intervention should lead to changes in the equilibrium wages, meaning that the partial equilibrium analysis potentially leaves out important margins.

Our paper explores the consequences of affirmative action in a simple general equilibrium framework. The main novelty compared with Coate and Loury (1993) is that wages are determined competitively and because of general equilibrium effects we obtain some results that are in sharp contrast with the analysis with exogenous wages concerning the effects of affirmative action.

We show that when a quota is introduced in a situation where blacks are discriminated against, the firms must respond by assigning more black (white) workers to the more (less) demanding job. This makes the expected productivity in the more demanding job (the job that requires human capital investment) lower for the marginal black worker than for the marginal white worker. The competitive wage is to pay a worker in the demanding job in accordance with expected marginal products (as without the quota), while workers in the low-skill job now are paid the expected marginal product in the demanding job for the critical worker in the group.

The quota thus tends to reduce the wage in the less demanding job for black workers. Contrary to the perverse effects on incentives in Coate and Loury's model this tends to *improve* incentives and while there may still be equilibria with discrimination under affirmative action the policy will at least result in a higher proportion of investors in the target group.[1] Instead, the policy can result in

[1] The caveat is that there may be several discriminatory equilibria and the result may not hold if workers initially coordinate on an equilibrium with an 'intermediate degree' of discrimination and re-coordinate on the most discriminatory equilibrium after the policy is introduced. To deal with this problem we compare worst case scenarios.

perverse distributional consequences. The improved incentives raise the proportion of skilled workers in the target group, but nothing guarantees that this effect is strong enough to offset the negative effect on wages in the low-skill job. Hence, it is possible that the targeted beneficiaries of affirmative action are made worse off by the policy.[2]

In addition to endogenizing wages we also consider a more general production technology than Coate and Loury (1993), which allows for different tasks to be complementary inputs in production. When tasks are complementary we show that incentives to invest for workers of any group are decreasing in the proportion of workers that invest in the other group. This cross-group externality is a novel feature in the literature on statistical discrimination and has interesting consequences for the willingness of a dominant group to eliminate discrimination. Unlike the standard models where discrimination is explained as a pure coordination failure, the dominant group will under plausible circumstances be better off under discrimination compared to the best symmetric equilibrium.

The rest of the paper is organized as follows. We present the model in Section 2. In Section 3 we characterize the equilibria in the laissez faire regime with the policy and explain briefly how our model is qualitatively different from the previous literature. The implications of labor market quotas are studied in Section 4 and in Section 5 we discuss how the labor market quota in our model corresponds with actual affirmative action policies. Section 6 concludes. Most proofs are relegated to Appendix A.

2. The model

The model has two firms engaged in Bertrand competition for workers and a continuum of workers with mass normalized to unity.[3] Each worker belongs to one of two identifiable groups, B or W and we denote by λ^J the respective fraction in the population for $J = B, W$. Prior to entering the labor market each worker must make an ex ante human capital investment decision. The decision is binary, either the worker invests in her human capital and becomes a *qualified* worker, or the

[2] Welch (1976) argues that some workers in the target group may be made worse off by a quota. Discussing a model where job assignments are first best efficient in the absence of a quota he argues that the marginal productivity in the unskilled job may go down if the firms would respond by assigning skilled whites to the unskilled job, thereby hurting unskilled blacks. Our argument is different in several ways. The driving force is that competitive wages are determined in a qualitatively different way with affirmative action and the logic does not rest on changes in the factor ratio. Moreover, *all* blacks may lose in our model.

[3] The model works also with a traditional competitive market with firms taking wages as given. This alternative model yields *exactly the same* equilibrium characterization. In this paper we have chosen to stick with an explicitly game theoretic formulation mainly because the economics of the model are easier to understand from the deviation arguments used in the analysis of Bertrand competition. In addition, some technical issues are avoided by the game theoretic model.

worker does not invest. If a worker invests, she incurs cost c which is distributed according to the cumulative distribution $G(c)$, while no cost is incurred otherwise. All agents are risk neutral, so the utility of a qualified worker who earns wage w is $w - c$, while the utility of a worker who does not invest is w.

To generate output, firms need workers performing two tasks, a *complex task* and a *simple task*. Only qualified workers are able to perform the complex task while all workers are equally productive in the simple task. The effective input of labor in the complex task, C, is thus taken to be the quantity of *qualified* workers employed in the complex task, while the input of labor in the simple task, S, is the quantity of workers (of both types) employed in the task. Output is given by $y(C, S)$ where y is a smooth neoclassical constant returns to scale production function.

Employers are unable to observe whether a worker is qualified or not, but observe instead a signal $\theta \in [0, 1]$, distributed according to density f_q if the worker is qualified and f_u otherwise. Both densities are continuously differentiable, bounded away from zero and $f_q(\theta)/f_u(\theta)$ is strictly increasing in θ. This monotone likelihood ratio property implies that the posterior probability that a worker from group J with signal θ is qualified given prior π^J,

$$p(\theta, \pi^J) \equiv \frac{\pi^J f_q(\theta)}{\pi^J f_q(\theta) + (1 - \pi^J) f_u(\theta)}, \tag{1}$$

is strictly increasing in θ, so qualified workers are more likely to get high signals. We let F_q and F_u denote the associated cumulative distributions.

The timing is described in Fig. 1. Firstly, each worker decides whether or not to invest and is then assigned a (publicly observable) signal θ by nature. The firms then simultaneously announce wages and workers' allocation to tasks, which are allowed to depend on the signal and group identity, so formally an action for firm i is to select some *wage schedule* $w_i^J: [0, 1] \rightarrow R_+$ and a *task assignment rule* $t_i^J: [0, 1] \rightarrow \{\text{complex, simple}\}$ for each group J. Workers observe the posted wages (and task assignment rules) and decide which firm to work for before payoffs are realized.

Since investments are not observed by the firms it is irrelevant whether wages are posted simultaneously or after the investment decisions.[4] The timing of events

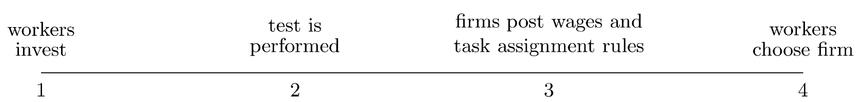

Fig. 1. Timeline (real time).

[4] See Moro and Norman (2001) for details.

should therefore be considered a description of how the game proceeds in 'real time', while in a game theoretic sense the posting of wages and task assignments is simultaneous with the investment decisions. The only place where sequential rationality plays a role in the analysis is therefore in the workers' acceptance rules.

3. Equilibria

We use as our solution concept Nash equilibria that satisfy the additional requirement that workers sort themselves to firms in a sequentially rational manner after any history of play. Formally, this means that we look for equilibria where conditionally strictly dominated strategies are eliminated.

Our first result describes the conditions on wages and task assignments that must hold in order for firms to behave optimally given some arbitrary investment behavior by the workers.

Proposition 1. *A necessary and sufficient condition for firms to play best responses when fractions* $\pi = (\pi^B, \pi^W)$ *of the workers invest is that both firms assign workers to tasks according to cut-off rules with thresholds* $(\theta^B(\pi), \theta^W(\pi))$ *which solve*

$$\max_{\theta^B, \theta^W \in [0,1]^2} y\Big(\sum_{J=B,W} \lambda^J \pi^J (1 - F_q(\theta^J)),\\ \sum_{J=B,W} \lambda^J (\pi^J F_q(\theta^J) + (1-\pi^J) F_u(\theta^J))\Big), \tag{2}$$

and post wage schedules $(w^b(\theta;\pi), w^W(\theta;\pi))$ *given by*[5]

$$w^J(\theta;\pi) = \begin{cases} \dfrac{\partial y}{\partial S} & \text{for } \theta < \theta^J(\pi) \\ p(\theta, \pi^J)\dfrac{\partial y}{\partial C} & \text{for } \theta \geq \theta^J(\pi) \end{cases} \qquad \text{for } J = B, W. \tag{3}$$

To understand this we first observe that the probability that a worker is qualified defined in (1) is strictly increasing in the signal, implying that cut-off rules are optimal when allocating workers to tasks. If all firms use the same threshold θ^J for group J (true in equilibrium) the inputs of complex and simple labor from group J are $\lambda^J \pi^J (1 - F_q(\theta^J))$ and $\lambda^J(\pi^J F_q(\theta^J) + (1-\pi^J)F_u(\theta^J))$, respectively, and total labor inputs are obtained by summing over the two groups. The proposition thus says that workers are allocated so as to maximize output conditional on investment

[5]Wages and task assignments can deviate from the characterization in Proposition 1 over sets of signals with measure zero. To be able to state succinctly if and only if results we ignore qualifications like 'for almost all θ' in the main body of the paper, but we do take care of this in the proofs.

behavior and that each worker is paid her expected marginal product in the task she is assigned to.

The omitted arguments of the partial derivatives in (3) are the arguments in (2) evaluated at the optimal solution. Since the labor inputs depend on both π^B and π^W this means that the equilibrium wage scheme for group J in general depends on the investment behavior in both groups. In order to stress this dependence (which is important for Proposition 3 below) we define the equilibrium factor ratio

$$r(\pi) = \frac{\sum_{J=B,W} \lambda^J \pi^J (1 - F_q(\theta^J(\pi))}{\sum_{J=B,W} \lambda^J (\pi^J F_q(\theta^J(\pi)) + (1-\pi^J) F_u(\theta^J(\pi)))}. \tag{4}$$

By constant returns we may evaluate the marginal products in (3) at $C = r(\pi)$ and $S = 1$.

The analysis is made considerably more straightforward from the fact that (2) has a unique solution. This means that the thresholds, and therefore also $r(\pi)$ is uniquely defined for each π.

Lemma 1. *Problem* (2) *has a unique solution whenever* $\pi^J > 0$ *for some* J.

In a full equilibrium, investments must be best responses to the wages, implying that a worker invests if and only if the gain in expected earnings is higher than the cost c. Given the best response wages, let $I^J(\pi)$ denote the (potentially group-specific) difference in expected earnings between investors and non-investors. We refer to this as the (gross) *incentives to invest* and use direct substitution from (3) to write this as

$$\begin{aligned} I^J(\pi) &= \int_0^1 w^J(\theta;\pi) f_q(\theta)\, \mathrm{d}\theta - \int_0^1 w^J(\theta;\pi) f_u(\theta)\, \mathrm{d}\theta \\ &= \frac{\partial y(r(\pi),1)}{\partial S} (F_q(\theta^J(\pi)) - F_u(\theta^J(\pi))) + \frac{\partial y(r(\pi),1)}{\partial C} \\ &\times \int_{\theta^J(\pi)}^1 p(\theta,\pi)(f_q(\theta) - f_u(\theta))\, \mathrm{d}\theta. \end{aligned} \tag{5}$$

The importance of (5) is that, by appeal to Lemma 1, it shows that conditional on any assumed investment behavior there is a uniquely determined benefit of investment.

In equilibrium, workers invest if and only if the benefit is higher than the cost, so $G(I^J(\pi))$ is the best response fraction of investment in group J and equilibria are characterized by a fixed point equation in π.

Proposition 2. *The fraction of investors in any equilibrium solves*

$$\pi^J = G(I^J(\pi)). \tag{6}$$

Moreover, any solution to (6) *corresponds with an equilibrium of the model.*

In light of this result we refer to a solution to (6) as an 'equilibrium' from now on. Equilibria always exist, which is established by checking continuity of the function on the right hand side of (6).[6]

Among the equilibria there is always at least one equilibrium where $\pi^B = \pi^W$. Employers have no incentives to treat groups differently if they invest at the same rate, so the thresholds solving (2) must be identical for the two groups. The wage schemes in (3) are thus identical across groups implying that incentives to invest are also the same. Symmetric equilibria are thus characterized by a univariate analogue to (6) and existence again follows from checking continuity.

A *discriminatory equilibrium* is a solution to (6) where $\pi^B \neq \pi^W$. Firms then correctly believe that workers from, say, group B are less likely to be qualified. In such an equilibrium the wage scheme for group B is uniformly below the wage schedule for group W and B workers receive lower signals on average, so average earnings are also lower in group B. Our notion of discrimination is therefore equivalent with the more conventional definition of economic discrimination in terms of wage differentials. Discriminatory equilibria exist for a non-negligible set of parametrizations of the model. The exact statement and proof of this claim is analogous to Proposition 5 in Section 4 and since our main focus is on the model with affirmative action we leave it to the reader to adapt the argument to the model without the policy.

3.1. Cross-group effects on incentives

A novel feature of our model compared to the literature is that there is a negative 'externality' *between groups*. In our model incentives to invest in human capital are affected adversely by an increase in human capital in the other group if $y(C, S)$ is strictly concave in both arguments. Moreover, the same forces that create these negative effects on incentives also make average income and welfare of one group to decrease with an increase in the human capital of the other group.

Proposition 3. *Suppose that* $y(C, S)$ *is homogenous of degree one and strictly concave in both arguments and fix* $\pi^B > 0$. *Then* $I^B(\pi^B, \pi^W)$ *is decreasing in* π^W *over the whole unit interval and strictly decreasing whenever* $0 < \theta^J(\pi) < 1$ *for both groups.*

[6] See Moro and Norman (2001) for details.

The reader may consult Moro and Norman (2001) for a complete proof. However, we use this result when comparing equilibria with and without affirmative action (Proposition 6 appeals to Proposition 3). Moreover, the proposition has some relevance in itself for how to rationalize color conscious policies. We will therefore provide a heuristic explanation of the result to help build some intuition for why Proposition 3 is true.

For simplicity, suppose that investments are not too different in the two groups, in which case (2) has an interior solution which is fully characterized by the first order conditions

$$p(\theta^J(\pi), \pi^J)\frac{\partial y(r(\pi), 1)}{\partial C} = \frac{\partial y(r(\pi), 1)}{\partial S} \tag{7}$$

for $J = B, W$. It is routine to check from (7) that the implicit function theorem applies so that the first order conditions may be differentiated with respect to π^J. The first step is to establish that $\partial r(\pi)/\partial \pi^J > 0$, that the ratio of complex to simple labor increases when more workers become qualified. To see this, assume for contradiction that $\partial r(\pi)/\partial \pi^J \leq 0$. The marginal productivity of complex (simple) labor is decreasing (increasing) in the factor ratio and $p(\theta, \pi^J)$ is increasing in both arguments so, by using (7), we see that if $\partial r(\pi)/\partial \pi^J \leq 0$, then $\partial \theta^J(\pi)/\partial \pi^J \leq 0$ for both groups with strict inequality in at least one group. We then note that $r(\pi)$ is defined in terms of π and $\theta^J(\pi)$ in (4) and that if $\partial \theta^J(\pi)/\partial \pi^J \leq 0$ for both groups, then (4) implies that $\partial r(\pi)/\partial \pi^W > 0$, which is a contradiction.

Hence, we conclude that $\partial r(\pi)/\partial \pi^W > 0$ and by again using condition (7) for group B it follows that $\partial \theta^B(\pi)/\partial \pi^W > 0$. The effect on the wage scheme for group B is shown in the graph to the left in Fig. 2, from which it should be clear that incentives to invest are reduced.

The increase in the factor ratio affect wages in the complex task negatively also

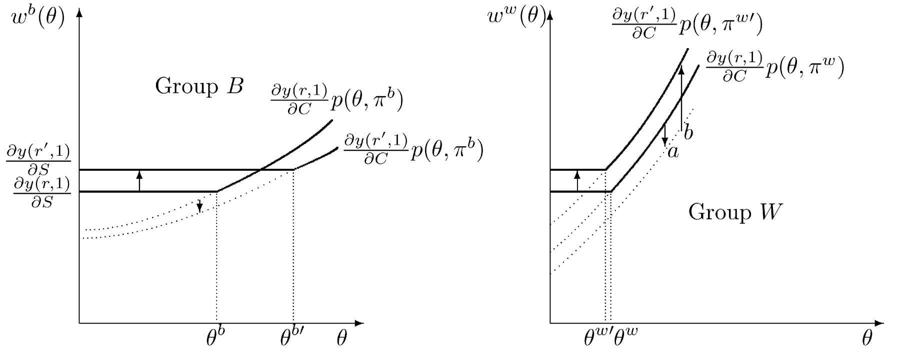

Fig. 2. The effect of an increase in the fraction of investors in group *W*.

in group W (see graph to the right in Fig. 2, $\downarrow_a$) but for that group there is also a direct effect $(\uparrow_b)$ due to the increase in the posterior probability that a worker from the group is qualified $(p(\cdot, \pi^W))$. The net effect on incentives can go either way in general and the only thing that can be shown is that for low enough π^W the effect is positive and that for high enough π^W the effect is negative.

The cross-group externality adds what we view as a rather realistic feature to the literature on equilibrium discrimination. A highly plausible possibility in our model is that all agents in the group with the higher fraction of qualified workers are better off in a discriminatory equilibrium than in the best symmetric equilibrium. The model is thus capable of explaining why a dominant majority would be hesitant to adopt policies aimed at eliminating discrimination, something that cannot be rationalized in standard models, where groups are treated as if they were living in separate economies (see for example Akerlof (1976), Coate and Loury (1993) and Spence (1974)). In these models discrimination is a pure *coordination failure* where some group is coordinating on a bad equilibrium and the dominant group would then have no reason to object if the coordination failure somehow could be solved for the discriminated group.

The externality between groups is driven by standard scarcity concerns and disappears in the special case where $y(C, S) = \alpha C + \beta S$ for some $\alpha > \beta > 0$ (the technology in Coate and Loury (1993) is of this form). The threshold signals solving (2) may then be obtained without reference to the other group since the marginal products are constant. Workers in each group earn $\alpha p(\theta, \pi^J)$ if the signal is above the threshold and β otherwise so the equilibrium conditions reduce to two identical fixed point equations in a single variable.

4. Affirmative action

Most real world affirmative action programs involve requirements that the representation of targeted groups is somehow comparable with the available pool of potential candidates. We follow Coate and Loury (1993) and model this as a constraint that says that *the share of workers from group B in each task* is equal to the share of workers from group B in the population. In equilibrium, all firms offer identical wage schedules and set identical thresholds and the affirmative action constraint for a representative firm then simply says that the proportion of workers below the threshold is the same in each group. In the affirmative action regime the constrained optimal threshold signals are therefore obtained as the solution to

$$\max_{\theta B,\ \theta W \in [0,\,1]^2} y\left(\sum_{J=B,W} \lambda^J \pi^J (1 - F_q(\theta^J)),\ \sum_{J=B,W} \lambda^J (\pi^J F_q(\theta^J) + (1 - \pi^J) F_u(\theta^J))\right)$$

$$\text{subject to}\quad \pi^B F_q(\theta^B) + (1 - \pi^B) F_u(\theta^B) = \pi^W F_q(\theta^W) + (1 - \pi^W) F_u(\theta^W). \tag{8}$$

While this cannot be seen from (8) it is actually important that there is affirmative action in both the complex and the simple task. See Section 5 for a brief discussion of the issues.

Our first result of the section characterizes equilibrium wages and task assignments.

Proposition 4. *A necessary and sufficient condition for firms to play best responses when fractions* $\pi = (\pi^B, \pi^W)$ *of the workers invest is that both firms assign workers to tasks according to cut-off rules with thresholds* $(\hat{\theta}^B(\pi), \hat{\theta}^W(\pi))$ *that are obtained as the* (*unique*) *solution to* (8) *and that all firms post wage schedules* $(\hat{w}^B(\theta;\pi), \hat{w}^W(\theta;\pi))$ *given by*

$$\hat{w}^J(\theta;\pi) = \begin{cases} p(\hat{\theta}^J(\pi), \pi^J)\dfrac{\partial y(\hat{r}(\pi),1)}{\partial C} & \text{for } \theta < \theta^J(\pi) \\ p(\theta, \pi^J)\dfrac{\partial y(\hat{r}(\pi),1)}{\partial C} & \text{for } \theta \geq \theta^J(\pi) \end{cases} \tag{9}$$

for $J = B, W$, *where the factor ratio* $\hat{r}(\pi)$ *is defined as* (4) *with thresholds replaced by* $(\hat{\theta}^B(\pi), \hat{\theta}^W(\pi))$.[7]

An outline of the proof is in Appendix A. For intuition we first observe that $\hat{\theta}^B(\pi) < \hat{\theta}^W(\pi)$ if $\pi^B < \pi^W$, which follows directly from the constraint in (8) after observing that F_q first order stochastically dominates F_u as a consequence of the monotone likelihood ratio assumption. The thresholds must also satisfy the necessary and sufficient conditions for a solution to (8),

$$\begin{aligned} &-\lambda^W \frac{\partial y(\hat{r}(\pi),1)}{\partial C}\, p(\hat{\theta}^W(\pi), \pi^W) + \lambda^W \frac{\partial y(\hat{r}(\pi),1)}{\partial S} - \mu = 0 \\ &-\lambda^B \frac{\partial y(\hat{r}(\pi),1)}{\partial C}\, p(\hat{\theta}^B(\pi), \pi^B) + \lambda^B \frac{\partial y(\hat{r}(\pi),1)}{\partial S} + \mu = 0 \end{aligned}. \tag{10}$$

Combining these two equations we have that the weighted average of the expected marginal productivity in the complex task for agents with the threshold signal equals the marginal productivity in the simple task, but since $p(\hat{\theta}^B(\pi), \pi^B) < p(\hat{\theta}^W(\pi), \pi^W)$ the expected marginal products for each group can be depicted as in Fig. 3 with a downwards jump at the threshold for group B and an upwards jump for group W. The graph illustrates that profitable 'within group' deviations are possible if workers are paid according to their expected marginal products. The

[7] Uniqueness of solutions to (8) is established by a simple variation of the argument in the proof of Lemma 1.

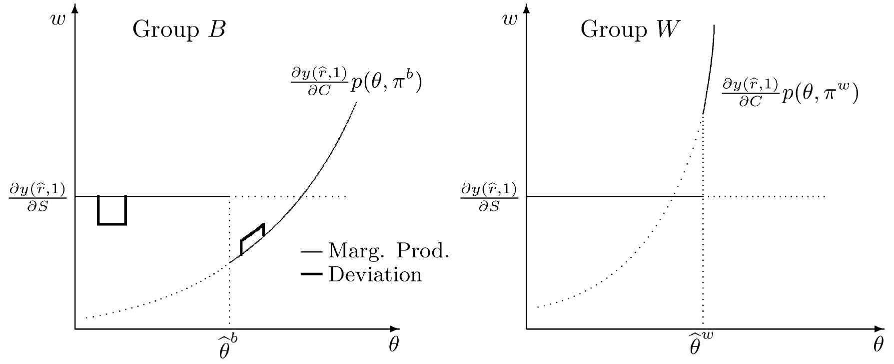

Fig. 3. A profitable deviation from paying marginal products.

proposed deviation is illustrated by the thicker lines in Fig. 3 and the idea is that a firm could raid the other firm(s) for low-paid workers currently in the complex task. By choosing the ranges for the deviation appropriately the quantity of workers from the group that are assigned to the simple task is held fixed, so output is kept constant and the affirmative action constraint is unaffected. Since wage payments are lower the deviation is profitable for the firm.

The equilibrium wage schedules are depicted in Fig. 4. While workers in the simple task are no longer paid their marginal products, the average payment is the marginal product, so firms break even in equilibrium. Intuitively it is as if the firm pays the marginal products to 'composite workers' consisting of a fraction λ^B of B-workers and a fraction λ^W of W-workers.

Given the equilibrium wages the rest of the equilibrium characterization is as without the policy. The incentives to invest are

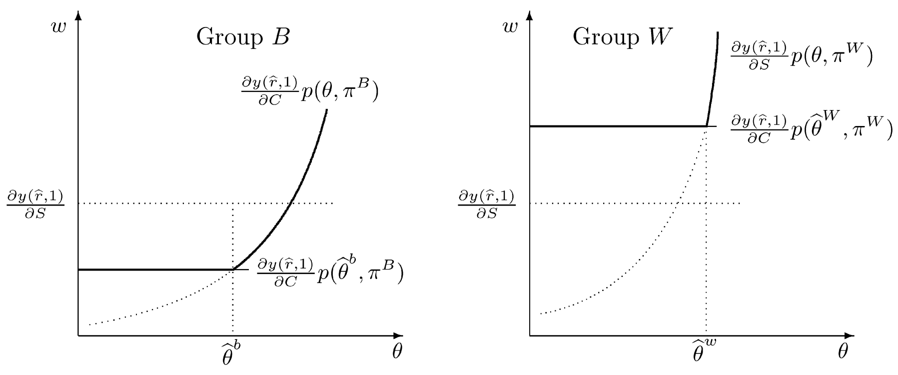

Fig. 4. Equilibrium wage schedules under affirmative action.

$$H^J(\pi) = \frac{\delta y(\hat{r}(\pi), 1)}{\partial C}\left[p(\hat{\theta}^J(\pi), \pi^J)(F_q(\hat{\theta}^J(\pi)) - F_u(\hat{\theta}^J(\pi)) + \int_{\hat{\theta}^J(\pi)}^{1} p(\theta, \pi^J)(f_q(\theta) - f_u(\theta))\, \mathrm{d}\theta \right], \tag{11}$$

and π is an equilibrium if and only if $\pi^J = G(H^J(\pi))$ for $J = B, W$. Any non-discriminatory equilibrium in the basic model is an equilibrium also under affirmative action since the unconstrained task assignment rules satisfy the affirmative action constraint when $\pi^B = \pi^W$. However, the symmetric equilibria exist with and without the policy so this simple fact is only interesting if the discriminatory equilibria are eliminated by affirmative action. In general this is not the case:

Proposition 5. *For any given y, f_q, f_u, λ^B and λ^W there exists a distribution function G such that the model with affirmative action has an equilibrium in which group B is discriminated.*

Proof. If $\pi^B = 0$ and $0 < \pi^W < 1$ we observe from (9) together with (11) that $H^B(0, \pi^W) = 0 < H^W(0, \pi^W)$. It is straightforward to verify that H^J is continuous at $(0, \pi^W)$ for $J = B, W$, so there exists $\pi^B > 0$ such that $0 < \pi^B < \pi^W < 1$ and (H^B is initially increasing) $0 < H^B(\pi^B, \pi^W) < H^W(\pi^B, \pi^W)$. It should be clear from Fig. 5 that one can find a strictly increasing function G such that $G(0) > 0$,

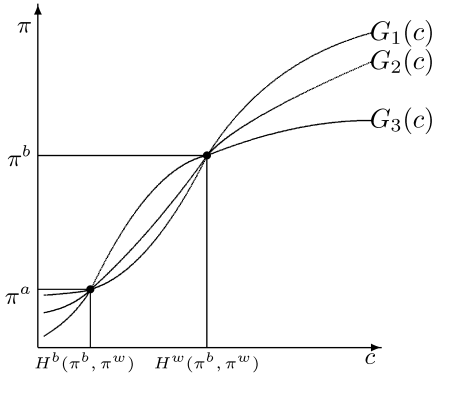

Fig. 5. Three distributions supporting given fractions of investors as an equilibrium.

$G(H^B(\pi^B, \pi^W)) = \pi^B$ and $G(H^W(\pi^B, \pi^W))) = \pi^W$, so that (π^B, π^W) is an equilibrium in the economy with fundamentals $\{y, f_q, f_u, (\lambda^B, \lambda^W), G\}$.[8] □

4.1. Affirmative action may make the discriminated group worse off

In our model, affirmative action may defeat the purpose of redistributing towards the discriminated group. The general intuition for this can be understood from Fig. 4. With affirmative action, the equilibrium wages are the solid lines and without affirmative action wages are given by marginal products. When the policy is introduced the wage is pushed down in the simple task; unless the factor ratio increases significantly it is then evident that the expected earnings for a worker in the discriminated group decrease. This effect may be moderated by increased investments, but if such response is small the workers in the discriminated group are made worse off by the policy.

For simplicity we consider a discrete cost distribution where there is a fraction β of workers with investment cost $c = 0$, a fraction γ with $c = \bar{c} > 0$ and a fraction $1 - \beta - \gamma$ with $c = \bar{\bar{c}}$, where $\bar{c} < \bar{\bar{c}}$ (see Fig. 6). The argument can be easily extended to strictly increasing distributions.

From the optimality conditions to (2) one shows that for any fixed $\pi^W > 0$ there exists $\epsilon > 0$ such that all workers from group B will be assigned to the simple task if $\pi^B < \epsilon$. Workers in group B then face a constant wage equal to the marginal product of labor in the simple task. Assuming that both types of labor are essential

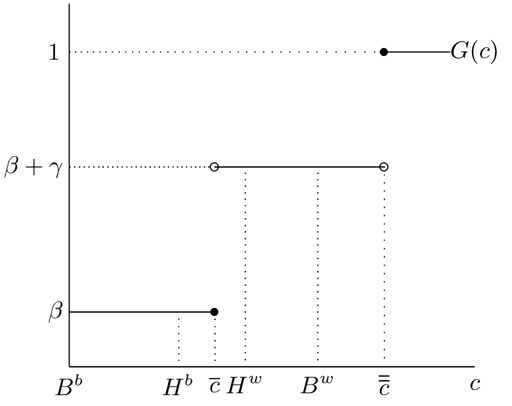

Fig. 6. A simple distribution that supports $(\pi^B, \pi^W) = (\beta, \beta + \gamma)$ as an equilibrium.

[8]To strengthen this to a genericity result one can show that any distribution G^* with $G^*(H^B(\pi^B, \pi^W)) < \pi^B$ and $G^*(H^W(\pi^B, \pi^W))) > \pi^W$ still admits an equilibrium with discrimination. The argument uses an extension of Proposition 3 to the model with affirmative action and implies that an open set of distributions (for example in sup-norm) admits a discriminatory equilibrium.

for production then some workers from group W must be assigned to the complex task. Hence, the wage for group W is strictly increasing in the signal above an interior threshold and translating this into incentives to invest this means that $I^B(\pi^B, \pi^W) = 0 < I^W(\pi^B, \pi^W)$ for π^B sufficiently small. Thus, if $\beta = \pi^B$ and $\gamma = \pi^W - \beta$ it follows that $(\pi^B, \pi^W) = (\beta, \beta + \gamma)$ is an equilibrium in the model without affirmative action given that $\bar{c}$ is in between 0 and $B^W(\pi^B, \pi^W)$. Moreover, for β small enough and for the right choice of $\bar{c}$ this is an equilibrium also with affirmative action. This follows since $H^B(\pi^B, \pi^W) \to 0$ as $\pi^B \to 0$ while $H^W(\pi^B, \pi^W) \to H^W(0, \pi^W) > 0$ as $\pi^B \to 0$ (see (10) and (9)). Hence there exists some $\beta > 0$ such that $0 = I^B(\beta, \beta + \gamma) < H^B(\beta, \beta + \gamma) < \min\{I^W(\beta, \beta + \gamma), H^W(\beta, \beta + \gamma)\}$. As is illustrated in Fig. 6 this implies that $(\pi^B, \pi^W) = (\beta, \beta + \gamma)$ is an equilibrium both with and without the affirmative action policy if $\bar{c}$ is chosen in between $H^B(\beta, \beta + \gamma)$ and the minimum of $I^W(\beta, \beta + \gamma)$ and $H^W(\beta, \beta + \gamma)$ and if $\bar{\bar{c}}$ is sufficiently large.

Without affirmative action, workers in group B are paid the marginal productivity in the simple task, $\partial y(r(\pi), 1)/\partial S$, which is strictly positive. Under affirmative action, $w^B(\theta) \leq \partial y(\hat{r}(\pi), 1)/\partial C \; p(1, \beta)$ for all θ. The right hand side of this inequality approaches zero when $\beta \to 0$, so for β small enough the policy decreases the expected utility for all agents in group B.

4.2. How are incentives changed by affirmative action?

Inspecting Fig. 4 it seems obvious that incentives to invest are strengthened (reduced) for the discriminated (dominant) group when the policy is introduced. Competitive wages under affirmative action are depicted by the thick lines, while without the policy workers would be paid $\partial y/\partial S$ up to the point where $\partial y/\partial S = p(\theta, \pi^J) \; \partial y/\partial C$ and then the expected marginal product in the complex task. However, this naive comparison of equilibrium wages with and without the policy may go wrong for several reasons: the factor ratio may change (for fixed investments), investment behavior changes and there are in general several equilibria with different degrees of group inequality both with and without the policy.

To deal with the multiplicity of equilibria we compare ‘worst case scenarios’. Defining the *most discriminatory* equilibrium as the equilibrium in which the difference in the fraction of investors is the largest[9] we can show that affirmative action improves incentives and increases the fraction of investors in the discriminated group if some regularity conditions hold and the discriminated group is small enough.

[9]One shows that such an equilibrium is also the equilibrium in which the fraction of investors in the discriminated (dominant) group is smallest (largest) by using Proposition 3 and its analogue for the model with affirmative action.

This result is trivial if the model allows an equilibrium with all agents in a group assigned to the simple task in the absence of affirmative action (then no qualifiers on group size are needed), but holds also when there is no such equilibrium, given that the 'direct effect' (holding investments constant) of the policy is to reduce the disparity in incentives. One sufficient condition for this is to assume that the discriminated group is a small enough fraction of the workforce, so that the change in the factor ratio is negligible.

Proposition 6. *Let* (π^B, π^W) *be the most discriminatory equilibrium in the model without affirmative action and let* $(\hat{\pi}^B, \hat{\pi}^W)$ *be the most discriminatory equilibrium in the model with affirmative action, where group B is the discriminated group in both cases. Then* $\pi^B < \hat{\pi}^B$ *if* λ^B *is sufficiently small.*

Complementarities between groups tend to strengthen the immediate effect of the policy, which is to improve incentives for the discriminated group and reduce them for the other group. The basic intuition may then be seen from Fig. 4. When λ^B is small the partial equilibrium effect of affirmative action on wages for group *B for fixed fractions of investment* is close to the naive comparison between paying marginal products and the affirmative action wages. The reason is that the affirmative action constraint will be met mainly by adjusting the threshold for the small group. Since the shadow cost of respecting the constraint becomes negligible as λ^B goes to zero, the effect on the factor ratio as well as the effect on equilibrium wages for the big group are negligible. One can then use the negative cross-group externality on incentives established in Proposition 3 to conclude that the general equilibrium effects tend to strengthen the partial equilibrium effect of removing affirmative action, implying that the fraction of investors in the discriminated group is higher with affirmative action when comparing the most discriminatory equilibrium under each regime.

There are other ways to guarantee that the effects due to changes in the factor ratio are small enough and Proposition 6 holds true for all of these. For example, if the production technology is linear, then the marginal productivities are independent of the factor ratio and the result holds irrespective of the relative size of the discriminated group.

4.3. Affirmative action may have the desired effect

In this example we show that it is indeed possible that affirmative action removes equilibria with discrimination and that the target group may be made better off. Suppose that a fraction π^* invests in *both groups*. Inspecting the equilibrium wages we see that the wage schedules then will be identical across groups in both cases (no reason to discriminate if the fraction of qualified workers is the same for both groups) and also the same across policy regimes (wages unaffected when the affirmative action constraint does not bind). The incentives to invest must therefore satisfy $I^B(\pi^*, \pi^*) = I^W(\pi^*, \pi^*) = H^W(\pi^*, \pi^*) = H^W(\pi^*, \pi^*)$. Now suppose that the cost function is of the same form as in the previous example and let $\pi^* = \beta + \gamma$ and again consider Fig. 6. Compared to the situation when $(\pi^B, \pi^W) = (\beta, \beta + \gamma)$ we know that the benefit of investment with both groups investing at rate $\beta + \gamma$ is somewhere in between H^B and I^W in the figure.[10] It is then immediate that if the cost distribution is changed so that $\underline{c}$ is moved to the left of H^B in the graph then $(\pi^B, \pi^W) = (\beta, \beta + \gamma)$ is no longer an equilibrium under the affirmative action regime, while it remains an equilibrium without affirmative action. With affirmative action the only remaining equilibrium is where a fraction $\beta + \gamma$ of the workers invest in each group and since agents in group B each has as a feasible option to invest as in the discriminatory equilibrium a simple revealed preference argument establishes that they are better off in the symmetric equilibrium.

5. Discussion: real world affirmative action and quotas

In the public debate on affirmative action, proponents often claim that they are in favor of affirmative action, but that they don't want quotas, and often even that the policy has nothing to do with quotas. One may therefore ask whether our analysis is of any relevance for this real world quota-free affirmative action policy that most proponents claim to be in favor of.

We view this part of the debate as mainly semantic. One interpretation of what is at stake is that proponents identify quotas with rigid numerical goals as opposed to more flexible ways of trying to increase representation of minorities in target areas. However, it is hard to understand what would be the distinction between affirmative action and simply trying to make sure that civil rights legislation isn't violated if there would be no numerical targets involved.[11] Moreover, there is ample evidence that courts are using statistical information as evidence for discrimination in a way that, assuming that possible defendants are rational and foresee this, creates incentives for use of numerical targets. The recent highly publicized cases of racial profiling in police work is an obvious example and there are lots of examples where promotion frequencies for different racial groups have been used as evidence of unequal treatment.

Hence, we do not view the fact that we model affirmative action as a quota as a shortcoming of the analysis. However, what arguably is a weakness is that we need to impose a quota in both tasks. If we would remove the quota in the simple task, the equilibrium would have to be immune against all the deviations that are

[10] This follows from Proposition 3 and the analogous result for the affirmative action regime, which is established in exactly the same way as Proposition 3.

[11] See Bergman (1996) for a more complete discussion of this. Arguing in favor of affirmative action, Bergman makes a rather convincing case that the policy must involve quotas to have any impact.

possible in the model with quotas in both tasks. For this reason, the equilibrium conditions in Proposition 4 are still necessary conditions for an equilibrium. But, with no quota in the simple task there is an obvious deviation — attract more workers from the cheap discriminated group and get rid of workers from the expensive group. We conclude from this that an equilibrium fails to exist, at least in pure strategies.

We concur that it would be nice to be able to handle a quota in the complex task only. Still, quotas in both tasks is not as outlandish as one may first think when interpreting the quotas as arising from a fear of lawsuits. Then evidence of the form that there is x% blacks in management, while other jobs have y% blacks may be bad for the firm, thereby effectively creating a quota also for lower end jobs.

6. Concluding remarks

This paper provides a framework where the effects of affirmative action on wages can be studied. Besides being theoretically more satisfactory than a model with exogenous wages, this makes it possible to study distributional consequences of affirmative action.

Our most striking finding is that affirmative action may *increase* the inequality between groups. The reason for this is that the partial equilibrium effect of affirmative action typically is to reduce the wage in the unskilled job for the discriminated group and increase the wage in the unskilled job for the other group. Changes in investment behavior tend to mitigate the partial equilibrium effects, but nothing guarantees that the response in terms of changes in human capital investments are large enough to reverse the initial effect.

The same effects on wages that create perverse distributional effects tend to create the desired effects on incentives. A color blind equilibrium is not guaranteed, but the partial equilibrium effect is typically to increase the incentives for the disadvantaged group. Comparisons between regimes are complicated by the multiplicity of equilibria, but comparing worst case scenarios we find that incentives are improved by affirmative action. Our analysis therefore suggests that one may want to worry less about the possibility of perverse effects of incentives and more about the possibility that affirmative action may harm the intended beneficiaries.

Acknowledgements

This paper contains parts of the material that was previously included in the paper ‘Affirmative Action in a Competitive Economy’, *SSRI Working Paper* No. 9914. Other parts from that paper are now included in ‘A General Equilibrium Model of Statistical Discrimination’. We are indebted to Stephen Coate, Stephen

Morris and Andrew Postlewaite for many long and helpful conversations and a number of valuable comments. Additionally, we want to acknowledge helpful discussions and comments from John Kennan, Narayana Kocherlakota, Marcos Lisboa, George Mailath, Lee Ohanian, Larry Samuelson and seminar participants at several universities. The usual disclaimer applies.

Appendix A. Proofs

While not necessary for the result we make the argument for two firms only and, when this simplifies the argument, we assume that for each θ where firms offer the same wage each firm gets a fraction 1/2 of the qualified as well as the unqualified workers with that signal.

In the proofs we adopt the convention that $t^J(\theta)$ is the fraction of workers with signal θ employed in the complex task. To economize on space we also define $f_{\pi^J}(\theta) \equiv \pi^J f_q(\theta) + (1-\pi^J) f_u(\theta)$ and $F_\pi(\theta) \equiv \pi F_q(\theta) + (1-\pi^J) F_u(\theta)$.

Proof of Proposition 1

A.1.1. Sufficiency

Proof. Let $(\theta^B(\pi), \theta^W(\pi))$ be a solution to problem (2) and for each J let t^J be the task-assignment rule with threshold $\theta^J(\pi)$ and $w^J(\theta; \pi)$ be defined by (3). Suppose one firm deviates from the strategy specified in Proposition 1 and plays an alternative strategy $\{w_d^B, w_d^W, t_d^B, t_d^W\}$. Also, let ϕ^J: $[0, 1] \rightarrow [0, 1]$ be the aggregate workers' acceptance rule, so that $\phi^J(\theta) \in [0, 1]$ is the fraction of the workers from group J with signal θ that picks the deviating firm given that the other firm sticks to the supposed equilibrium strategy. We immediately note that sequential rationality on behalf of the workers implies that $w_d^J(\theta) \geq w^J(\theta; \pi)$ for all θ such that $\phi^J(\theta) > 0$, implying that the total wage costs for the deviating firm must satisfy

$$W_d = \sum_J \lambda^J \int \phi^J(\theta) w_d^J(\theta) f_{\pi^J}(\theta)\, \mathrm{d}\theta \geq \sum_J \lambda^J \int \phi^J(\theta) w^J(\theta, \pi) f_{\pi^J}(\theta)\, \mathrm{d}\theta$$

$$\geq \sum_J \lambda^J \int_{\theta \in [0,1]} \big(1 - t_d^J(\theta)\big)\phi^J(\theta) \frac{\partial y(r(\pi), 1)}{\partial S} f_{\pi^J}(\theta)\, \mathrm{d}\theta \tag{A.1}$$

$$+ \sum_J \lambda^J \int_{\theta \in [0,1]} t_d^J(\theta)\phi^J(\theta) \frac{\partial y(r(\pi), 1)}{\partial C} p(\theta, \pi^J) f_{\pi^J}(\theta)\, \mathrm{d}\theta$$

$$= \frac{\partial y(r(\pi), 1)}{\partial S} S_d + \frac{\partial y(r(\pi), 1)}{\partial C} C_d, \tag{A.2}$$

where the second inequality uses that $w^J(\theta;\ \pi) = \max\{\partial y(r(\pi),\ 1)/\partial S,\ \partial y(r(\pi),\ 1)/\partial C\ p(\theta,\ \pi^J)\}$ and where

$$S_d = \sum_J \lambda^J \int_{\theta \in [0,1]} \left(1 - t_d^J(\theta)\right) \phi^J(\theta) f_{\pi^J}(\theta)\, \mathrm{d}\theta \text{ and}$$

$$C_d = \sum_J \lambda^J \int_{\theta \in [0,1]} t_d^J(\theta) \phi^J(\theta) p\left(\theta,\ \pi^J\right) f_{\pi^J}(\theta)\, \mathrm{d}\theta$$

$$= \sum_J \lambda^J \int_{\theta \in [0,1]} t_d^J(\theta) \phi^J(\theta) \pi^J f_q(\theta)\, \mathrm{d}\theta \tag{A.3}$$

are the effective factor inputs in the two tasks under the deviation. We now let C and S denote the factor inputs for the deviating firm in the proposed equilibrium and note that the profit from the deviation satisfies

$$\Pi_d = y(C_d, S_d) - W_d \leq y(C, S) + \frac{\partial y(C, S)}{\partial S}\left(S^d - S\right) + \frac{\partial y(C, S)}{\partial C}\left(C^d - C\right) - W_d$$

$$= \frac{\partial y(r(\pi), 1)}{\partial S} S^d + \frac{\partial y(r(\pi), 1)}{\partial C} C^d - W_d = 0, \tag{A.4}$$

by using homogeneity of degree zero in the first derivatives and Eulers' theorem. Hence, the deviation is not profitable and since the deviation was arbitrary it follows that the conditions in Proposition 1 are sufficient for an equilibrium. □

A.1.2. Necessity

Proof. We proceed by successively ruling out different possibilities. Since complete deviation arguments are heavy in notation we only sketch some of the most intuitive steps, but complete proofs of these steps are available in Moro and Norman (1999).

Step 1. The first step is to verify that any equilibrium task assignment rule must be characterized by a threshold for each group, where a worker is assigned to the complex task if and only if the signal is above the group-specific threshold. The argument uses the strict monotone likelihood property of $f_q(\theta)/f_u(\theta)$ to establish that any rule that deviates from a threshold rule on a measurable subset of [0, 1] is dominated by a threshold rule, since it is possible to increase the effective input of labor in both tasks when task assignments are not following a threshold rule.

Step 2. Next, one establishes that both firms must offer the same wages to both groups for almost all θ. The reason is that the firm that offers the higher wages over some set with positive measure could deviate by offering, say, the average wage. This change wouldn't affect the distribution of available workers, so the firm could keep output constant and reduce wages, a profitable deviation (there is a subtle issue ignored in this seemingly obvious argument that is discussed in Step 3 below).

Step 3. In this proof we will simply assert that since wages are the same almost everywhere, each firm faces a distribution of workers that is a scaling of the population distribution. One way to make sense of this is to build into the model that tie-breaking rules cannot be conditioned on whether the worker is qualified or not (there are more fundamental rationalizations of this). The more purist view is to allow workers to break ties arbitrarily, except that the aggregate rules must continue to be measurable. This would for example allow qualified workers to break ties in favor of firm 1 and unqualified workers to break ties in favor of firm 2. However, it is intuitively plausible that tie-breaking rules that generate a favorable selection for one firm over an interval couldn't be part of an equilibrium. This intuition is right and the reader can consult Moro and Norman (2001) for a detailed proof that shows that no equilibrium can have workers breaking ties in a way so that the way ties are broken provides any information to the firms.

Step 4. Given that each firm faces a scaling of the population distribution each firm faces a problem which is equivalent with (2) when deciding how to optimally assign workers to tasks, implying that any equilibrium task assignments must be in accordance to a solution to this problem. This completes the proof of the necessity of the equilibrium task assignment rules in Proposition 1.

Step 5. It remains to show that equilibrium wages must be in accordance to (3). The first step in this argument is to show that equilibrium wages must satisfy natural arbitrage conditions guaranteeing that the price of an 'effective unit' of labor is independent of the signals in the range where workers are assigned to a particular signal. To show this in the simple task, we need to demonstrate that there exists some k_S^J such that $w_i^J(\theta) = k_S^J$ for (almost) all $\theta \le \theta^J(\pi)$. For contradiction, suppose that there exist some k, some $\delta > 0$ and sets Θ_L, $\Theta_H \subseteq [0, \theta^J(\pi)]$ with positive measure such that $w_i^J(\theta) \le k - \delta$ for all $\theta \in \Theta_L$ and $w_i^J(\theta) - k \ge 0$ for all $\theta \in \Theta_H$ for group J. Consider a deviation $\{w_d^B, w_d^W, t_d^B, t_d^W\}$ by firm i where

$$w_d^J(\theta) = \begin{cases} w_i^J(\theta) + \epsilon & \text{for } \theta \in \Theta_L \\ 0 & \text{for } \theta \in \Theta_H. \\ w_i^J(\theta) & \text{otherwise} \end{cases} \tag{A.5}$$

For simplicity we assume that each firm gets 1/2 of the workers with each signal. We can handle other possibilities, but only with some additional notation. By continuity of f_{π^J} we may w.l.o.g. assume $\int_{\theta \in \Theta_L} f_{\pi^J}(\theta)\,\mathrm{d}\theta = \int_{\theta \in \Theta_H} f_{\pi^J}(\theta)\,\mathrm{d}\theta > 0$, which implies (given that half of the workers is in each firm) that the input of both factors remains constant if task assignments are unchanged, which we assume. The change in the profit is thus just the negative of the change in wage payments,

$$\Delta\Pi(\epsilon) = \frac{1}{2}\left[\int_{\theta \in \Theta_H} w_i^J(\theta) f_{\pi^J}(\theta)\,\mathrm{d}\theta - \int_{\theta \in \Theta_L} \left(w_i^J(\theta) + 2\epsilon\right) f_{\pi^J}(\theta)\,\mathrm{d}\theta\right]$$
$$\geq \left(\frac{\delta}{2} - c\right) \int_{\theta \in \Theta_L} f_{\pi^J}(\theta)\,\mathrm{d}\theta \tag{A.6}$$

Since $\lim_{\epsilon \to 0} \Delta\Pi(\epsilon) > 0$ the deviation is profitable for ϵ small enough.

Step 6. In the complex task the situation is similar, but the wage must be equal 'per efficiency unit' meaning that it should be proportional to the probability of being qualified. Hence, there exists k_C^J such that $w_i^J(\theta) = p(\theta, \pi)k_C^J$ for almost all $\theta \in [\theta^J(\pi), 1]$. The proof mimics the argument used in Step 5.

Step 7. Next we show that the wage scheme must be continuous for each group. If task assignments are in a corner solution for a group this follows from Steps 5 and 6, so we need to establish that $k_S^J = p(\theta^J(\pi), \pi^J)k_C^J$ whenever $\theta^J(\pi) \in (0, 1)$. Again the proof is by contradiction and exploits the discontinuity to generate a profitable deviation. Suppose $k_S^J > p(\theta^J(\pi), \pi^J)k_C^J$. We construct a deviation such that output is unchanged but with a lower wage bill implying higher profits. For example consider θ^* small enough so that $p(\theta^*, \pi^J)k_C^J < k_S^J$:

$$w_d^J(\theta) = \begin{cases} 0 & \text{for } \theta \in [0, \theta') \\ w^J(\theta) + \epsilon & \text{for } \theta \in [\theta^J(\pi), \theta^*) \\ w^J(\theta) & \text{otherwise} \end{cases}$$
$$t_d^J(\theta) = \begin{cases} 0 & \text{for } \theta \in [0, \theta'') \\ 1 & \text{for } \theta \in [\theta'', 1] \end{cases}. \tag{A.7}$$

where θ' solve $F_{\pi^J}(\theta') = F_{\pi^J}(\theta^*) - F_{\pi^J}(\theta^J(\pi))$ and θ'' solve $F_{\pi^J}(\theta'') - F_{\pi^J}(\theta^J(\pi)) = (F_{\pi^J}(\theta^*) - F_{\pi^J}(\theta^J(\pi)))/2$. The definition of θ' implies that the mass of workers hired by the firm is unchanged and θ'' is set so that the input of simple labor is unchanged. The change in the input of complex labor under the deviation is $\pi^J(F_q(\theta^*) - 2F_q(\theta'') + F_q(\theta^J(\pi)))/2$ and some algebra shows that this change is positive. Thus, output increases and the difference in profits must be larger than the reduction in wage costs which is

$$\Delta\Pi(\epsilon) = \frac{1}{2} k_S^J F_{\pi^J}(\theta') - \frac{1}{2} \int_{\theta^J(\pi)}^{\theta^*} p\left(\theta, \pi^J\right)k_C^J f_{\pi^J}(\theta)\,\mathrm{d}\theta - \int_{\theta^J(\pi)}^{\theta^*} \epsilon f_{\pi^J}(\theta)\,\mathrm{d}\theta. \tag{A.8}$$

The first term in $\Delta\Pi(\epsilon)$ is the wage saving for not hiring workers with $\theta \in [0, \theta']$. The second is the additional wages paid for attracting workers with $\theta \in [\theta^J(\pi), \theta^*]$ from the other firms. Recall that $p(\theta, \pi^J)k_C^J < k_S^J$ for $\theta \in (\theta^J, \theta^*)$ and $F_{\pi^J}(\theta') = F_{\pi^J}(\theta^*) - F_{\pi^J}(\theta^J)$. Thus, $k_S^J F_{\pi^J}(\theta') = \int_{\theta^J}^{\theta^*(\pi)} k_S^J f_{\pi^J}(\theta)\ \mathrm{d}\theta/2 > \int_{\theta^J}^{\theta^*(\pi)} p(\theta, \pi^J)k_C^J f_{\pi^J}(\theta)\ \mathrm{d}\theta/2$, so $\lim_{\epsilon\to 0} \Delta\Pi(\epsilon) > 0$, implying that there are $\epsilon > 0$ such that the deviation is profitable. The case with $k_S^J > p(\theta^J(\pi), \pi^J)k_C^J$ can be treated symmetrically.

Step 8. It remains to show that $k_S^J = \partial y(r(\pi), 1)/\partial S$ and $k_C^J = \partial y(r(\pi), 1)/\partial C$ for each group J. To see this let $C^J = \pi^J(1 - F_q(\theta^J(\pi)))$ and $S^J = F_{\pi^J}(\theta^J(\pi))$ be the per capita input of labor attributable to group J in each task and note that constant returns allows to express output in the economy as

$$y(C, S) = \sum_J \lambda^J \left[\frac{\partial y(r(\pi), 1)}{\partial C} C^J + \frac{\partial y(r(\pi), 1)}{\partial S} S^J \right]. \tag{A.9}$$

Total wage costs (summed over the firms) in the economy are

$$\begin{aligned} W &= \sum_J \lambda^J \left[\int_\theta w^J(\theta) f_{\pi^J}(\theta)\ \mathrm{d}\theta \right] \\ &= \sum_J \lambda^J \left[\int_0^{\theta^J(\pi)} k_S^J f_{\pi^J}(\theta)\ \mathrm{d}\theta + \int_{\theta^J(\pi)}^1 k_C^J p(\theta, \pi^J) f_{\pi^J}(\theta)\ \mathrm{d}\theta \right] \\ &= \sum_J \lambda^J [k_S^J S^J + k_C^J C^J] \end{aligned} \tag{A.10}$$

Hence the profit for a firm is a scaling of

$$\Pi = \sum_J \lambda^J \left[\left(\frac{\partial y(r(\pi), 1)}{\partial C} - k_C^J \right) C^J + \left(\frac{\partial y(r(\pi), 1)}{\partial S} - k_S^J \right) S^J \right], \tag{A.11}$$

which means that a failure of the result would mean that a firm would have an incentive to change either C^J or S^J, contradicting the optimality of the task assignment rules which was established in Step 4. □

A.2. Proof of Lemma 1

Proof. Transform problem (2) to the equivalent problem

$$\begin{aligned} &\max_{\{C^J, S^J\}_{J=B,W}} y(\lambda^W C^W + \lambda^B C^B, \lambda^W S^W + \lambda^B S^B) \\ &\text{subject to. } 0 \le H(C^J, S^J; \pi^J) \\ &\quad = \pi^J - C^J - S^J + (1 - \pi^J) F_u\left(F_q^{-1}\left(\frac{\pi^J - C^J}{\pi^J} \right) \right) \\ &\quad \text{for } J = B, W \end{aligned} \tag{A.12}$$

Using the monotone likelihood ratio one shows that H is concave, so the constraint set is convex. Strict quasi-concavity of the objective thus guarantees that the solution must be unique. The one exception is (potentially) if $\pi^B = \pi^W = 0$ in which case $C^B = C^W = 0$ in any feasible solution. If $y(0, S) = 0$ for all $S > 0$ there will then be (trivial) multiplicity. □

A.3. Proof of Proposition 4

A.3.1. Sufficiency

Proof. Consider a deviation $\{w_d^B, w_d^W, t_d^B, t_d^W\}$. Let ϕ^J:[0, 1]→[0, 1] be the aggregate workers' acceptance rule given a unilateral deviation from the assumed equilibrium, so that $\phi^J(\theta) \in [0, 1]$ is the fraction of the workers from group J with signal θ that picks the deviating firm given that the other firm sticks to the supposed equilibrium strategy. We also let C, S and C_d, S_d denote the factor inputs in the candidate equilibrium and under the deviation, respectively. Due to constant returns we have that $\partial y(C, S)/\partial C = \partial y(\hat{r}(\pi), 1)/\partial C$ and sequential rationality on behalf of the workers implies that $w_d^J(\theta) \geq \hat{w}^J(\theta; \pi^J) \geq p(\theta, \pi^J)\ \partial y(\hat{r}(\pi), 1)/\partial C$ for all $\theta \in [\hat{\theta}^J(\pi), 1]$ such that $\phi^J(\theta) > 0$. Total wage payments to workers employed in the complex task under the deviation, W_d^C are thus

$$\begin{aligned} W_d^C &= \sum_{J=B,W} \left[\int_\theta t_d^J(\theta)\phi^J(\theta) w_d^J(\theta) f_\pi(\theta)\, \mathrm{d}\theta \right] \\ &\geq \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \sum_{J=B,W} \left[\int_\theta t_d^J(\theta)\phi^J(\theta) p(\theta, \pi^J) f_\pi(\theta)\, \mathrm{d}\theta \right] \\ &= \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \sum_{J=B,W} \left[\int_\theta t_d^J(\theta)\phi^J(\theta)\pi^J f_q(\theta)\, \mathrm{d}\theta \right] = \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} C_d \end{aligned} \quad \text{(A.13)}$$

Similarly, for workers employed in the simple task after the deviation we have that $w_d^J(\theta) \geq \hat{w}^J(\theta; \pi) \geq p(\hat{\theta}^J(\pi), \pi^J)(\partial y(\hat{r}(\pi),1)/\partial C)$, so total wage payments *to workers in the simple task under the deviation are*

$$\begin{aligned} W_d^S &= \sum_{J=B,W} \left[\int_\theta (1 - t_d^J(\theta))\phi^J(\theta) w_d^J(\theta) f_\pi(\theta)\, \mathrm{d}\theta \right] \\ &\geq \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \sum_{J=B,W} \left[\int_\theta p(\hat{\theta}^J(\pi), \pi^J)(1 - t_d^J(\theta))\phi^J(\theta) f_\pi(\theta)\, \mathrm{d}\theta \right] \\ &= \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \sum_{J=B,W} p(\hat{\theta}^J(\pi), \pi^J) S_d^J, \end{aligned} \quad \text{(A.14)}$$

where S_d^J is the mass of group J workers employed in the simple task. The deviation must respect the affirmative action constraint which implies that $S_d^J = \lambda^J S_d$, so

$$W_d^S \geq \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \sum_{J=B,W} p(\hat{\theta}^J(\pi), \pi^J) S_d^J = S_d \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} \times \sum_{J=B,W} p(\hat{\theta}^J(\pi), \pi^J)\lambda^J = S_d \frac{\partial y(\hat{r}(\pi), 1)}{\partial S}, \quad \text{(A.15)}$$

where the last equality comes from the first order condition (10). The profits under the deviation are thus

$$\Pi_d \leq y(C_d, S_d) - \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} C_d - \frac{\partial y(\hat{r}(\pi), 1)}{\partial C} S_d \leq 0, \quad \text{(A.16)}$$

where the last inequality comes from using concavity and constant returns on $y(C_d, S_d)$ exactly in the same way as in the necessity part in the end of the proof of Proposition 1.

A.3.2. Necessity

The proof proceeds ruling out different possibilities as the proof of the necessity part in Proposition 1.

Steps 1–5. These steps are identical as in the proof of Proposition 1 and therefore omitted.

Step 6. We could get to the point where we know that both firms post the same wages and that $\hat{w}^J(\theta) = k_S^J$ over $[0, \hat{\theta}^J(\pi)]$ for some constant k_S^J for each group by replicating the same arguments as in the proof of Proposition 1. The reason is that all these arguments leave the affirmative action constraint unaffected. As in the model without the policy we will now show that $\hat{w}^J(\theta) = p(\theta, \pi^J)k_C^J$ for some constant k_C^J for θ, but the argument used in Step 6 in the proof of Proposition 1 *cannot* be applied here because deviation (A.5) in general affects the affirmative action constraint. We therefore proceed by constructing a deviation that keeps the affirmative action constraint satisfied.

We prove the argument for continuous wage schemes (since measurable functions can be well approximated by continuous functions, the argument can be extended to arbitrary measurable wage functions, but it involves some tedious approximations and is therefore omitted; the complete proof is available from the authors).

Suppose that w_i^J are continuous, which implies that $w_1^J = w_2^J = w^J$ exactly in any equilibrium. If the claim is false for one group, which we without loss take to be group B, then $w^B(\cdot)/p(\cdot, \pi^B)$ must be either decreasing or increasing at some point $\theta^{*B} > \hat{\theta}^B$ which means that there is some scalar h^B and intervals $(\underline{\theta}_h^B, \overline{\theta}_h^B)$, $(\underline{\theta}_l^B, \overline{\theta}_l^B)$ such that $w^B(\theta)/p(\theta, \pi^B) > h^B > w^B(\theta')/p(\theta', \pi^B)$ for each $\theta \in (\underline{\theta}_h^B, \overline{\theta}_h^B)$, $\theta' \in (\underline{\theta}_l^B, \overline{\theta}_l^B)$. Similarly, there will also be some point $\theta^{*W} > \hat{\theta}^W$ and some scalar h^W such that there are intervals $(\underline{\theta}_h^W, \bar{\theta}_h^W)$, $(\underline{\theta}_l^W, \bar{\theta}_l^W)$ such that $w^W(\theta)/p(\theta, \pi^W) \geq h^W \geq w^W(\theta')/p(\theta', \pi^W)$ for each $\theta \in (\underline{\theta}_h^W, \bar{\theta}_h^W)$, $\theta' \in (\underline{\theta}_l^W, \bar{\theta}_l^W)$. Furthermore, we may construct the intervals such that $\int_{\underline{\theta}_h^B}^{\bar{\theta}_h^B} f_{\pi^B}(\theta)\, d\theta = \int_{\underline{\theta}_h^W}^{\bar{\theta}_h^W} f_{\pi^W}(\theta)\, d\theta$ and $\int_{\underline{\theta}_l^B}^{\bar{\theta}_l^B} f_{\pi^B}(\theta)\, d\theta = \int_{\underline{\theta}_l^W}^{\bar{\theta}_l^W} f_{\pi^W}(\theta)\, d\theta$ and, which is important, we may without loss assume that either $\bar{\theta}_h^J = \underline{\theta}_l^J = \theta^{J*}$ or $\bar{\theta}_l^J = \underline{\theta}_h^J = \theta^{J*}$ (depending on whether $w^J(\cdot)/p(\cdot, \pi^J)$ is decreasing or increasing). First consider the deviation

$$w_{\text{dev}}^J(\theta) = \begin{cases} 0 & \text{if } \theta \in (\underline{\theta}_h^J, \bar{\theta}_h^J) \\ w^J(\theta) & \text{otherwise} \end{cases} \quad J = B, W. \tag{A.17}$$

By construction of the intervals, these wages satisfy affirmative action if the task assignments are unchanged relative to the candidate equilibrium (cut-off rule with thresholds $\hat{\theta}$). Letting $\Delta C^J = -\lambda^J \int_{\underline{\theta}_h^J}^{\bar{\theta}_h^J} \pi^J f_q(\theta)\, d\theta$ (literally, the right hand side should be divided by 2, but 1/2 would appear in all terms), $\Delta C = \Delta C^B + \Delta C^W$, C and S be the initial factor inputs, the change in profits may be expressed as $\Delta\pi = y(C + \Delta C, S) - y(C, S) - \lambda^J \int_{\underline{\theta}_h^J}^{\bar{\theta}_h^J} w^J(\theta) f_{\pi^J}(\theta)\, d\theta$. Dividing by $|\Delta C|$ we get that

$$\begin{aligned} \frac{\Delta\pi}{|\Delta C|} &= \frac{y(C + \Delta C, S) - y(C, S)}{|\Delta C|} + \frac{\sum_J \lambda^J \int_{\underline{\theta}_h^J}^{\bar{\theta}_h^J} w^J(\theta) f_{\pi^J}(\theta)\, d\theta}{|\Delta C|} \\ &> \frac{y(C + \Delta C, S) - y(C, S)}{|\Delta C|} + \frac{\sum_J \lambda^J \int_{\underline{\theta}_h^J}^{\bar{\theta}_h^J} h^J p(\theta, \pi^J) f_{\pi^J}(\theta)\, d\theta}{|\Delta C|} \\ &= \frac{y(C + \Delta C, S) - y(C, S)}{|\Delta C|} + \frac{\sum_J h^J |\Delta C^J|}{|\Delta C|}, \end{aligned} \tag{A.18}$$

where we have used that $p(\theta, \pi^J) f_{\pi^J}(\theta) = \pi^J f_q(\theta)$. If the limit of integration which corresponds to θ^{J*} is kept fixed for each group and the other limits (the upper when $w^J(\cdot)/p(\cdot, \pi^J)$ is increasing and the lower when $w^J(\cdot)/p(\cdot, \pi^J)$ is decreasing) approach θ^{J*} in such a way so that the affirmative action constraint holds along the sequence we can use l'Hopitals rule to verify that

$$\frac{|\Delta C^B|}{|\Delta C|} \to \frac{\lambda^B \pi^B f_q(\theta^{B*})}{\lambda^B \pi^B f_q(\theta^{B*}) + \lambda^W \pi^W f_q(\theta^{W*}) \dfrac{f_{\pi B}(\theta^{*B})}{f_{\pi B}(\theta^{*W})}} \equiv f^*$$

and that

$$\lim_{\Delta C \to 0} \frac{\Delta\pi}{\Delta C} > -y_1(C, S) + h^B f^* + h^W(1 - f^*). \tag{A.19}$$

Next, we instead consider the 'opposite' deviation where instead of firing high paid workers the firm steal low paid workers from the other firm,

$$w^J_{\mathrm{dev}}(\theta)=\begin{cases} w^J(\theta)+\delta & \text{if } \theta\in(\underline{\theta}^J_l,\overline{\theta}^J_l) \\ w^J(\theta) & \text{otherwise}\end{cases} J=B,W. \tag{A.20}$$

We can repeat the same steps as above to get that $\Delta C^B/\Delta C\to f^*$ as $\overline{\theta}^J_h\to\underline{\theta}^J_h$ in such a way so that the affirmative action constraint holds along the sequence and that

$$\begin{aligned}\lim_{\Delta C\to 0}\frac{\Delta\pi}{\Delta C} &> y_1(C,S)-h^Bf^*-h^W(1-f^*) \\ &\quad -\delta\left(\frac{\lambda^Bf_{\pi^B}(\underline{\theta}^B_l)+\lambda^Wf_{\pi^W}(\underline{\theta}^W_l)\dfrac{f_{\pi^B}(\underline{\theta}^B_l)}{f_{\pi^W}(\underline{\theta}^W_l)}}{\lambda^B\pi^Bf_q(\underline{\theta}^B_l)+\lambda^W\pi^Wf_q(\underline{\theta}^W_l)\dfrac{f_{\pi^B}(\underline{\theta}^B_l)}{f_{\pi^W}(\underline{\theta}^W_l)}}\right) \\ &\to y_1(C,S)-h^Bf^*-h^W(1-f^*) \text{ as } \delta\to 0.\end{aligned} \tag{A.21}$$

Combining (A.19) and (A.21) we have that if neither deviation is profitable for any $\overline{\theta}^B_h$ $(\overline{\theta}^B_l)>0$, then $-y_1(C,S)+h^Bf^*+h^W(1-f^*)<0$ and $y_1(C,S)-h^Bf^*-h^W(1-f^*)<0$, which is a contradiction.

Step 7. Wages must be continuous at $\hat{\theta}^J(\pi)$ for each group, which is established using a deviation argument as that in the main text when we show that expected marginal products are no longer consistent with equilibrium.

Step 8. We now argue that $k^J_C=\partial y(\hat{r}(\pi),1)/\partial C$ in any equilibrium. The key idea here is that there is a difference between the quantity of workers hired and their effective input of labor. That is, if one group is paid below the expected marginal productivity and the other above (if both groups are either under or overpaid it would pay to reallocate labor to or from the simple task) it would pay to deviate in such a way that only workers with relatively low signals would be attracted from the overpaid group and only workers with relatively high signals would be attracted from the underpaid group. Such a deviation increases the effective input of labor from the cheap group and decreases it from the expensive group and is profitable due to the fact that wages are proportional to $p(\theta,\pi^J)$.

Using Steps 1–6, the total wage costs for workers from group J is $k^J_C[p(\hat{\theta}^J,\pi^J)F_{\pi^J}(\hat{\theta}^J)+1-F_q(\hat{\theta}^J)]$, which is strictly increasing in k^J_C. To show that a wage schedule with $k^J_C\neq y_1(C,S)$ cannot be an equilibrium, we suppose that $k^B_C>y_1(C,S)$, which by zero profits implies $k^W_C<y_1(C,S)$. We will construct a deviation that deals only with workers assigned to the complex task, building on the observation that the cost of labor per productive worker is higher in group B than in group W. We substitute high test workers in group B with low test workers (as in Step 6)

and keep the number of workers employed in the skilled task constant. This reduces the input of qualified labor from group B. Doing exactly the opposite in the other group W restores the original quantity of qualified workers. Furthermore, the total wage costs are lower than in the candidate equilibrium while output is kept constant and the affirmative action constraint is still satisfied. Formally, consider the following deviation:

$$w_d^B(\theta) = \begin{cases} w_i^B(\theta) + \epsilon & \text{for } \theta \in [\hat{\theta}^B, \theta^{B\prime}] \\ 0 & \text{for } \theta \in [\theta^{B*}, 1] \\ w_i^B(\theta) & \text{otherwise} \end{cases}$$

$$w_d^W(\theta) = \begin{cases} 0 & \text{for } \theta \in [\hat{\theta}^W, \theta^{W\prime}] \\ w_i^W(\theta) + \epsilon & \text{for } \theta \in [\theta^{W*}, 1] \\ w_i^W(\theta) & \text{otherwise} \end{cases} \tag{A.22}$$

where $\theta^{B\prime}$, θ^{B*}, $\theta^{W\prime}$ and, θ^{W*} satisfy $\hat{\theta}^J < \theta^{J\prime} < \theta^{J*} < 1$, $J = B, W$ and are defined as follows. Firstly, $\theta^{B\prime}$ and θ^{B*} satisfy $F_{\pi^B}(\theta^{B\prime}) - F_{\pi^B}(\hat{\theta}^B) = 1 - F_{\pi^B}(\theta^{B*})$. Secondly, $\theta^{W\prime}$ and θ^{W*} satisfy the following equations:

$$F_{\pi^W}(\theta^{W\prime}) - F_{\pi^W}(\hat{\theta}^W) = 1 - F_{\pi^W}(\theta^{W*})$$
$$\times \int_{\theta^{W*}}^{1} \pi^W f_q(\theta) - \int_{\hat{\theta}^W}^{\theta^{B\prime}} \pi^W f_q(\theta) = \frac{\lambda^B}{\lambda^W} \int_{\theta^{B*}}^{1} \pi^B f_q(\theta) - \int_{\hat{\theta}^B}^{\theta^{B\prime}} \pi^B f_q(\theta) \tag{A.23}$$

If $\theta^{B\prime}$ is close enough to $\hat{\theta}^B$ a solution to (A.23) exists. The first equation guarantees that the number of employed workers remains constant, and the second that the effective input of qualified workers and therefore also output is constant. Thus, the change in profits is the change in wage costs,

$$\begin{aligned} w(\epsilon) - w = {} & \frac{\lambda^B}{2} \int_{\hat{\theta}^B}^{\theta^{B\prime}} \left(w_i^B(\theta) + \epsilon\right) f_{\pi^B}(\theta)\, \mathrm{d}\theta - \frac{\lambda^B}{2} \int_{\theta^{B*}}^{1} w_i^B(\theta) f_{\pi^B}(\theta)\, \mathrm{d}\theta \\ & - \frac{\lambda^W}{2} \int_{\hat{\theta}^W}^{\theta^{W\prime}} w_i^W(\theta) f_{\pi^W}(\theta)\, \mathrm{d}\theta + \frac{\lambda^W}{2} \int_{\theta^{W*}}^{1} \left(w_i^W(\theta) + \epsilon\right) f_{\pi^W}(\theta)\, \mathrm{d}\theta \\ = {} & \left(k_C^W - k_C^B\right)\Psi^B + \frac{\epsilon}{2} \left[\lambda^B \int_{\hat{\theta}^B}^{\theta^{B\prime}} \pi^B f_q(\theta)\, \mathrm{d}\theta + \lambda^W \int_{\theta^{W*}}^{1} \pi^W f_q(\theta)\, \mathrm{d}\theta \right], \end{aligned} \tag{A.24}$$

where $\Psi^B = \lambda^B/2\left(\int_{\theta^{B*}}^{1} \pi^B f_q(\theta) - \int_{\hat{\theta}^B}^{\theta^{B\prime}} \pi^B f_q(\theta)\right) < 0$ is the loss of complex labor from group B ($-\Psi^B$ is the additional input from group W). Since $k_C^W < y_1(C, S) < k_C^B$ and $\Psi^B < 0$ the deviation is profitable for ϵ small enough. □

A.4. Proof of Proposition 6

Proof. Let λ^B be small enough so that $I^B(\hat{\pi}^B, \hat{\pi}^W) < H^B(\hat{\pi}^B, \hat{\pi}^W)$. Now for $J = B, W$ let $\pi_0^J = \hat{\pi}^J$ and construct the sequences $\{\pi_t^J\}_{t=0}^{\infty}$ by letting π_t^B be the smallest solution to $\pi_t^B = G(I^B(\pi_t^B, \pi_{t-1}^W))$ and π_t^W be the largest solution to $\pi_t^W = G(I^B(\pi_{t-1}^B, \pi_t^W))$ for $t = 1, 2, \ldots$. For $t = 1$ one verifies that $\pi_1^B < \pi_0^B$ by using the intermediate value theorem ($G(I^B(\hat{\pi}^B, \hat{\pi}^W))$ is below the diagonal and $G(0)$ is above). On the other hand π_1^W may be larger or smaller than π_0^W. Hence for the next step we could potentially get a problem (we want to construct monotone sequences). However, if we instead let $\pi_0^W = \hat{\pi}^W - \epsilon$ it follows by continuity that $\pi_1^B < \pi_0^B$ for ϵ sufficiently small and for λ^B small enough we get $\pi_1^W > \pi_0^W$. For $t > 1$ we see that if $\pi_t^W > \pi_{t-1}^W$ then it follows from Proposition 3 that $G(I^B(\pi_t^B, \pi_t^W)) < G(I^B(\pi_t^B, \pi_{t-1}^W)) = \pi_t^B$ and since $G(I^B(0, \pi_t^W)) = 0$ we can again apply the intermediate value theorem to conclude that $\pi_{t+1}^B < \pi_t^B$ (we are presuming that $\pi_t^B > G(0)$: if for some t $\pi_t^B = G(0)$, then the process stops and we have reached a corner equilibrium, a case where the result holds trivially). Similarly, we have that if $\pi_t^B < \pi_{t-1}^B$ then $\pi_{t+1}^W > \pi_t^W$. Hence, $\{\pi_t^B\}$ is a monotonically decreasing sequence and $\{\pi_t^W\}$ is a monotonically increasing sequence. Since $\pi_t^J \in [0, 1]$ for each t it follows that both sequences are converging to some limits (π^{B*}, π^{W*}) in $[0, 1]$. Clearly, (π^{B*}, π^{W*}) is an equilibrium of the model without policy and since $\pi^{B*} < \hat{\pi}^B$ the result follows. □

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