A General Equilibrium Model of Statistical Disrcimination - Omitted Appendices

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In this document we provide calculations and proofs that for the sake of brevity have been omitted from the paper.

B Omitted Proofs

B.1 How to make individual and aggregate distributions coincide

Feldman and Gilles discuss alternative ways to ensure that the individuals' probability distribution and the frequency distribution coincides almost surely. The analysis in this paper relies only on this property and not the particular way we make sure that the property holds. The simplest solution is to use "aggregate shocks" rather than to assume that the signals are i.i.d. draws from F_q and F_u. Let H_q and H_u be the distributions of qualified and unqualified workers on $[\underline{c},\overline{c}]$ and x be uniformly distributed on $[0,1]$.The test-score for a qualified agent c, $\theta_c(x)$, is then taken to be

$$\theta_c(x) = \begin{cases} F_q^{-1}(H_q(c)+x) & \text{if } H_q(c)+x \leq 1 \\ F_q^{-1}(H_q(c)+x-1) & \text{if } H_q(c)+x > 1 \end{cases}$$

It is straightforward, but somewhat tedious to verify that $\Pr[\theta_c(x) \leq \theta \mid e_q] = F_q(\theta)$ for all $c \in [\underline{c},\overline{c}]$ and all $\theta \in [0,1]$ and that $\int\limits_{c\in A(x,\theta)} dH_q(c) = F_q(\theta)$ $x \in [0,1]$ and all $\theta' \in [0,1]$, where $A(x,\theta) = \{c \in [\underline{c},\overline{c}] \mid \theta_c(x) > \theta\}$. Clearly the construction can be applied to the unqualified agents as well.

B.2 Proof of Lemma 1.

Lemma 1 *Suppose that either 1) y is quasi-concave and strictly increasing in both arguments and $\frac{f_q(\theta)}{f_u(\theta)}$ is strictly increasing in θ, or, 2) y is strictly quasi-concave and strictly increasing in both arguments. Then, there is a unique $(\theta^B(\pi),\theta^W(\pi)) \in [0,1]^2$ that solves (8) for any $\pi >> 0$.*

Proof. Given any $\theta \in [0,1]$, let $C = \pi(1-F_q(\theta))$ and $S = \pi F_q(\theta) + (1-\pi)F_u(\theta)$. Since F_q is invertible any $C \in [0,\pi]$ can be achieved by the uniquely defined signal $\theta = F_q^{-1}\left(\frac{\pi - C}{\pi}\right)$, and the maximal level of S given C is thus

$$S = H(C) \equiv \pi - C + (1-\pi)F_u\left(F_q^{-1}\left(\frac{\pi - C}{\pi}\right)\right). \tag{B1}$$

We may thus restate the maximization problem in (3) as

$$\max_{C^B,C^W \in [0,\pi^B]\times[0,\pi^W]} y\left(\sum \lambda^j C^j, \sum \lambda^j H(C^j)\right). \tag{B2}$$

Differentiating $H(C)$ we get

$$H'(C) = -\frac{\pi f_q\left(F_q^{-1}\left(\frac{\pi - C}{\pi}\right)\right) + (1-\pi) f_u\left(F_q^{-1}\left(\frac{\pi - C}{\pi}\right)\right)}{\pi f_q\left(F_q^{-1}\left(\frac{\pi - C}{\pi}\right)\right)} = -\frac{1}{p(F_q^{-1}\left(\frac{\pi - C}{\pi}\right), \pi)} \tag{B3}$$

Now, $p(F_q^{-1}\left(\frac{\pi - C}{\pi}\right), \pi)$ is strictly decreasing in C since p is strictly increasing in the first argument and F_q^{-1} is strictly increasing (inverse of increasing function). Hence $H'(C)$ is strictly decreasing, so H is strictly concave. For contradiction, suppose $C^* = (C^{B*}, C^{W*})$ and $C^{**} = (C^{B**}, C^{W**})$ both solve (B2). Without loss, assume that $C^{B*} \neq C^{B**}$, and, for $\gamma \in (0,1)$, let $C^{B\gamma}$ denote $\gamma C^{B*} + (1-\gamma) C^{B**}$. By strict concavity of H we have that $H\left(C^{B\gamma}\right) > \gamma H\left(C^{B*}\right) + (1-\gamma) H\left(C^{B**}\right)$. Since y is strictly increasing in both arguments this implies that

$$y\left(C', \lambda^B H\left(C^{B\gamma}\right) + \lambda^W H\left(C^{\gamma}\right)\right) > y\left(C', \lambda^B\left(\gamma H\left(C^{B*}\right) + (1-\gamma) H\left(C^{B**}\right)\right) + \lambda^W H\left(C^{\gamma}\right)\right), \tag{B4}$$

where $C' = \lambda^B C^{B\gamma} + \lambda^W C^W$. Let $C^{W\gamma}$ be defines analogously with $C^{B\gamma}$ and note that the inequality in (B4) holds weakly (i.e., equality if $C^{B*} = C^{B**}$), which means that

$$\begin{aligned} y\left(\sum \lambda^J C^{J\gamma}, \sum \lambda^J H\left(C^{J\gamma}\right)\right) &> y\left(\sum \lambda^J C^{J\gamma}, \sum \lambda^J\left(\gamma H\left(C^{J*}\right) + (1-\gamma) H\left(C^{J**}\right)\right)\right) \\ &\geq \min\left\{y\left(\sum \lambda^J C^{J*}, \sum \lambda^j H\left(C^{J*}\right)\right), y\left(\sum \lambda^J C^{J**}, \sum \lambda^J H\left(C^{J**}\right)\right)\right\}, \end{aligned} \tag{B5}$$

by quasi-concavity, contradicting the hypothesis that C^* and C^{**} were optimal. ■

B.3 Proof of Lemma A1

Lemma A1 *Each firm earns a zero profit in any equilibrium*

Proof. Let Π_1 and Π_2 denote the profits for firm 1 and 2 and assume for contradiction that $\Pi_1 > 0$. If $\Pi_2 < 0$ there would be a profitable deviation, so we assume without loss that $0 \leq \Pi_2 \leq \Pi_1$.Total *industry* profits are

$$\Pi_1 + \Pi_2 = y(C_1, S_1) + y(C_2, S_2) - \int_\theta \max\{w_1(\theta), w_2(\theta)\}(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta. \tag{B6}$$

We observe that $y(C_1, S_1) + y(C_2, S_2) \leq y(C(\pi), S(\pi))$where $C(\pi)$ and $S(\pi)$ are factor inputs solving (3), which simply means that aggregate output can't exceed what a planner could achieve. Suppose firm 2 deviates by offering w_2' given by $w_2'(\theta) = \max\{w_1(\theta), w_2(\theta)\} + \epsilon$ for some $\epsilon > 0$, implying that firm 2 attracts all workers. In addition, suppose firm 2 assign as in the solution to (3). The corresponding profit is

$$\begin{aligned} \Pi_2' &= y(C(\pi), S(\pi)) - \int_\theta \max\{w_1(\theta), w_2(\theta)\}(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta - \epsilon \\ &= y(C(\pi), S(\pi)) - y(C_1, S_1) - y(C_2, S_2) + \Pi_1 + \Pi_2 - \epsilon \geq \Pi_1 + \Pi_2 - \epsilon. \end{aligned} \tag{B7}$$

Hence, for ϵ sufficiently small $\Pi_2' > \Pi_2$, a profitable deviation. ■

B.4 Proof of Lemma A2

Lemma A2 $w_1(\theta) = w_2(\theta)$ *for almost all* $\theta \in [0,1]$ *in any equilibrium*

Proof. For contradiction, suppose $w_1(\theta) > w_2(\theta)$ for all $\theta \in \Theta' \subset [0,1]$ and let

$$\delta = \int_{\theta \in \Theta'} (w_1(\theta) - w_2(\theta))(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta, \tag{B8}$$

where Θ' has positive measure, which implies that $\delta > 0$. Let C_1, S_1, C_2, S_2 be the effective factor inputs in the hypothetical equilibrium and suppose firm 1 deviates and offers w_1' given by $w_1'(\theta) = w_2(\theta) + \epsilon$ for all θ and assigns all workers (the deviation attracts all workers) in accordance with a solution to (3). The implied profits are

$$\begin{aligned} & \Pi_1'(\epsilon) = y(C(\pi), S(\pi)) - \int_\theta w_2(\theta)(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta - \epsilon \qquad \text{(B9)} \\ = \ & y(C(\pi), S(\pi)) - \int_{\theta \in \Theta'} w_1(\theta)(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta + \delta \\ & - \int_{\theta \in [0,1] \setminus \Theta'} w_2(\theta)(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta - \epsilon \\ \geq \ & y(C_1, S_1) + y(C_2, S_2) - \int_\theta \max\{w_1(\theta), w_2(\theta)\}(\pi f_q(\theta) + (1-\pi) f_u(\theta)) d\theta + \delta - \epsilon \\ = \ & \delta - \varepsilon, \end{aligned}$$

where the last inequality follows from Lemma A1. The deviation is thus profitable if ϵ is small enough. ■

B.5 Proof of Proposition 4

Proposition 4. *Suppose* y *satisfies assumptions A1-A6. Then there is always at least one symmetric equilibrium*

Proof. Let $\theta(\pi)$ denote the (by Lemma 1) unique solution to the problem (3) and $r(\pi)$ the corresponding factor ratio (defined as 19) with $\pi^B = \pi^W = \pi$ and $\theta^B(\pi) = \theta^W(\pi) = \theta(\pi)$. We observe that the unique solution to (3) $\theta(\pi)$ must be interior and satisfy

$$p(\theta(\pi), \pi) \frac{\partial y(r(\pi), 1)}{\partial C} = \frac{\partial y(r(\pi), 1)}{\partial S}, \tag{B10}$$

given any $\pi > 0$. Define $\rho : (0,1] \times [0,1] \to R_+$ by $\rho(\theta,\pi) \equiv \frac{\pi(1-F_q(\theta))}{\pi F_q(\theta)+(1-\pi)F_u(\theta)}$. By constant returns we may write the necessary condition for an interior solution to (3) as

$$D(\theta,\pi) = -p(\theta,\pi)\frac{\partial y(\rho(\theta,\pi),1)}{\partial C} + \frac{\partial y(\rho(\theta,\pi),1)}{\partial S} = 0 \tag{B11}$$

Straightforward differentiation establishes that $\frac{\partial\rho(\theta,\pi)}{\partial\theta} < 0$ (F_q and F_u are both strictly increasing), so one can check that $\frac{\partial D(\theta,\pi)}{\partial\theta} < 0$ by a direct computation. Hence, the implicit function theorem applies establishing that $\theta(\pi)$ is continuously differentiable in a neighborhood of $\theta(\pi^*)$ for any given $\pi^* > 0$. Hence, $\theta(\pi)$ is continuously differentiable and therefore continuous over $(0,1)$, implying that (6) is continuous over $(0,1)$. Remains to verify is that continuity holds at the boundaries as well. For $\pi = 1$ we have that $\frac{\partial y(r(1),1)}{\partial C} = \frac{\partial y(r(1),1)}{\partial S}$ from the first order condition to (3) and the wage is consequently a constant function of θ, so $I(1) = 0$. It is a relatively simple exercise to verify that $\lim_{\pi\to1} I(\pi) = 0$ and we leave this to the reader. For $\pi = 0$ any θ solves the problem (3), but output is zero in any solution and $w(\theta,0) = 0$ for all θ, so $I(0) = 0$. The first order condition (B10) must hold for any $\pi > 0$, which implies that $r(\pi) \to 0$ as $\pi \to 0$ since otherwise $\frac{\partial y(r(\pi),1)}{\partial S}$ would converge to something strictly positive and $p(\theta(\pi),\pi)\frac{\partial y(r(\pi),1)}{\partial C}$ would converge to zero. Assumptions A3, A5 and A6 imply that $\lim_{C\to0}\frac{\partial y(C,S)}{\partial S} = 0$ for any $S > 0$ and, again using (B10), this implies that $\lim_{\pi\to0} p(\theta(\pi),\pi)\frac{\partial y(r(\pi),1)}{\partial C} = 0$. Since $\frac{f_q(\theta)}{f_u(\theta)}$ is strictly increasing there exists a unique signal $\theta^* \in (0,1)$ such that $f_u(\theta^*) = f_q(\theta^*)$ (observe that f_q and f_u are densities so $f_u(\theta) <(>) f_q(\theta)$ for θ sufficiently high (low) in order for both densities to integrate to unity).

Now,

$$\begin{aligned}
I(\pi) &= \frac{\partial y(r(\pi),1)}{\partial S}\underbrace{(F_q(\theta(\pi)) - F_u(\theta(\pi)))}_{-} + \frac{\partial y(r(\pi),1)}{\partial C}\int_{\theta(\pi)}^1 p(\theta,\pi)(f_q(\theta) - f_u(\theta))d\theta < \qquad \text{(B12)}\\
&< \frac{\partial y(r(\pi),1)}{\partial C}\int_{\theta(\pi)}^1 p(\theta,\pi)(f_q(\theta) - f_u(\theta))d\theta < \frac{\partial y(r(\pi),1)}{\partial C}\int_{\theta^*}^1 p(\theta,\pi)(f_q(\theta) - f_u(\theta))d\theta < \\
&< \frac{\partial y(r(\pi),1)}{\partial C}p(1,\pi)(F_u(\theta^*) - F_q(\theta^*)) = \frac{\partial y(r(\pi),1)}{\partial C}p(\theta(\pi),\pi)\frac{p(1,\pi)}{p(\theta(\pi),\pi)}(F_u(\theta^*) - F_q(\theta^*)) \\
&\to 0 \text{ as } \pi \to 0,
\end{aligned}$$

since $\frac{\partial y(r(\pi),1)}{\partial C}p(\theta(\pi),\pi) \to 0$ and $\frac{p(1,\pi)}{p(\theta(\pi),\pi)}$ is bounded. Hence, I is continuous over the whole interval $[0,1]$ and existence follows by the intermediate value theorem. ■

B.6 Proof of Lemma A5

Lemma A5 $r(\pi)$ *is increasing in both arguments and strictly increasing in* π^J *for each* π *such that* $\theta^J(\pi) > 0$.

Proof. The Kuhn-Tucker conditions to the program (8) are necessary and sufficient for a solution and these conditions may after some rearrangements be written as

$$\begin{aligned} -p\left(\theta^J(\pi),\pi^J\right)\frac{\partial y(r(\pi),1)}{\partial C}+\frac{\partial y(r(\pi),1)}{\partial S}+\gamma^J-\kappa^J &= 0 \qquad \text{for } J=B,W, \qquad \text{(B13)}\\ \gamma^J\theta^J(\pi) = 0, \quad \kappa^J\left(1-\theta^J(\pi)\right) = 0, \quad \gamma^J \geq 0, \quad \kappa^J &\geq 0, \end{aligned}$$

where γ^J and κ^J are positive scalars of the multipliers to the constraints $\theta^J \geq 0$ and $1-\theta^J \geq 0$. Without loss, assume that $\pi^B < \pi^{B\prime}$ and $r(\pi) = r(\pi^B,\pi^W) > r(\pi^{B\prime},\pi^W) = r(\pi')$. We then claim that (B13) implies that $\theta^J(\pi) \geq \theta^J(\pi')$ for $J=B,W$. To see this, observe that if $\theta^J(\pi) < \theta^J(\pi')$, then since $\pi^J \leq \pi^{J\prime}$ for each J it follows that $p\left(\theta^J(\pi),\pi^J\right) < p\left(\theta^J(\pi'),\pi^{J\prime}\right)$, so

$$\begin{aligned} \kappa^J-\gamma^J &= -p\left(\theta^J(\pi),\pi^J\right)\frac{\partial y(r(\pi),1)}{\partial C}+\frac{\partial y(r(\pi),1)}{\partial S} > \qquad \text{(B14)}\\ &> -p\left(\theta^J(\pi'),\pi^{J\prime}\right)\frac{\partial y(r(\pi),1)}{\partial C}+\frac{\partial y(r(\pi),1)}{\partial S} > /r(\pi) > r(\pi')/\\ &\geq -p\left(\theta^J(\pi'),\pi^{J\prime}\right)\frac{\partial y(r(\pi'),1)}{\partial C}+\frac{\partial y(r(\pi'),1)}{\partial S} = \kappa^{J\prime}-\gamma^{J\prime}, \end{aligned}$$

where the two equalities come from the fact that the Kuhn-Tucker conditions in (B13) must be satisfies at π and π'. But $\theta^J(\pi) < \theta^J(\pi') \Rightarrow \theta^J(\pi) < 1$ and $\theta^J(\pi') > 0$, so $\kappa^J = 0$ and $\gamma^{J\prime} = 0$, which means that $-\gamma^J > \kappa^{J\prime}$ from (B14), which is a contradiction since the multipliers must be positive. Hence $\theta^J(\pi) \geq \theta^J(\pi')$, implying $\pi^j\left(1-F_q\left(\theta^J(\pi)\right)\right) \leq \pi^j\left(1-F_q\left(\theta^J(\pi)\right)\right)$ and

$$\begin{aligned} \pi^J F_q(\theta^J(\pi))+(1-\pi^J)F_u(\theta^J(\pi))) &\geq \pi^J F_q(\theta^J(\pi'))+(1-\pi^J)F_u(\theta^J(\pi'))) \qquad \text{(B15)}\\ /F_q \text{ first order stochatically dominates } F_u/ &\geq \pi^{J\prime}F_q(\theta^J(\pi'))+(1-\pi^{J\prime})F_u(\theta^J(\pi'))), \end{aligned}$$

Using the definition of $r(\pi)$ in (19) this means that $r(\pi) \leq r(\pi')$, a contradiction. Hence $r(\pi) \geq r(\pi')$ as claimed. Finally, to see that $r(\pi)$ is strictly increasing when $\theta^B(\pi) > 0$ we observe that if $r(\pi) \geq r(\pi')$ the exact same argument as above can be used to establish that $\theta^J(\pi) \geq \theta^J(\pi')$ for $J=B,W$. Moreover (B15) continues to hold for each group and *strictly so for group* B *since* $\pi^B < \pi^{B\prime}$. Hence we conclude that if $r(\pi) \geq r(\pi')$, then $r(\pi) < r(\pi')$, a contradiction. ■

B.7 Proof of Lemma A6

Proof. In the range where solutions are interior, optimal task assignments are fully characterized by a pair of first order conditions,

$$p\left(\theta^J(\pi),\pi^J\right)-\frac{y_2\left(R(\theta(\pi),\pi,1)\right)}{y_1\left(R(\theta(\pi),\pi,1)\right)} = 0 \qquad \text{(B16)}$$

for $J = B, W$, where $R(\theta, \pi) = \frac{\sum_{J=B,W} \lambda^J \pi^J (1-F_q(\theta^J))}{\sum_{J=B,W}^{J} F_{\pi^J}(\theta^j)}$ and the associated Jacobian matrix with respect to the endogenous variables is

$$J = \begin{pmatrix} \frac{\partial p(\theta^B(\pi),\pi^B)}{\partial \theta^B} - \frac{\partial}{\partial \theta^B}\left[\frac{y_2(R(\theta(\pi),\pi,1)}{y_1(R(\theta(\pi),\pi,1)}\right] & -\frac{\partial}{\partial \theta^W}\left[\frac{y_2(R(\theta(\pi),\pi,1)}{y_1(R(\theta(\pi),\pi,1)}\right] \\ -\frac{\partial}{\partial \theta^B}\left[\frac{y_2(R(\theta(\pi),\pi,1)}{y_1(R(\theta(\pi),\pi,1)}\right] & \frac{\partial p(\theta^W(\pi),\pi^W)}{\partial \theta^W} - \frac{\partial}{\partial \theta^W}\left[\frac{y_2(R(\theta(\pi),\pi,1)}{y_1(R(\theta(\pi),\pi,1)}\right] \end{pmatrix}. \tag{B17}$$

J is clearly invertible ($\frac{\partial p(\theta^J(\pi),\pi^J)}{\partial \theta^J} > 0$), so the conclusion that θ is differentiable follows as a consequence of the Implicit Function Theorem. ■.

C Calculations for the Parametric Example

We let $y(C, S) = C^\alpha S^{1-\alpha}$ for some $\alpha \in (0, 1)$ and assume that θ is drawn from $\{\theta_L, \theta_H\}$ in accordance with symmetric conditional probability distributions, where $\phi > \frac{1}{2}$ is the probability of drawing θ_H for a qualified worker, and $(1 - \phi)$ is the probability of θ_H for an unqualified worker.

In an equilibrium with equal treatment of groups, let σ and γ denote the fraction of agents assigned to the complex task with signal θ_H and θ_L respectively. The associated inputs of labor are

$$\begin{aligned} C(\sigma, \gamma, \pi) &= \sigma\phi\pi + \gamma(1-\phi)\pi \\ S(\sigma, \gamma, \pi) &= (1-\sigma)\left[\phi\pi + (1-\phi)(1-\pi)\right] + (1-\gamma)\left[(1-\phi)\pi + \phi(1-\pi)\right], \end{aligned} \tag{C18}$$

where π is the (common) fraction of investors. Optimal task assignments, denoted $(\sigma(\pi), \gamma(\pi))$, solve

$$\max_{\sigma,\gamma,\in[0,1]^2} C(\sigma, \gamma, \pi)^\alpha S(\sigma, \gamma, \pi)^{1-\alpha}, \tag{C19}$$

and closed form solutions to (C19) are easy to find from the first order conditions to the problem. This setup is equivalent to one with continuous signals with conditional distributions satisfying the monotone likelihood ratio property *weakly*, a case covered by Propositions 1 and 2. The unique continuation equilibrium wages are thus the expected marginal products, that is

$$w(\theta; \pi) = \max\left\{ p(\theta, \pi)\, \alpha \frac{S(\sigma(\pi), \gamma(\pi), \pi)}{C(\sigma(\pi), \gamma(\pi), \pi)}^{1-\alpha}, (1-\alpha)\, \frac{C(\sigma(\pi), \gamma(\pi), \pi)}{S(\sigma(\pi), \gamma(\pi), \pi)}^{\alpha} \right\}, \tag{C20}$$

for $\theta \in \{\theta_H, \theta_L\}$. Hence, the closed form solution to (C19) enables us to express the incentives to invest,

$$I(\pi) = \underbrace{\phi w(\theta_H;\pi) + (1-\phi)w(\theta_L;\pi)}_{\text{Exp wage for qualified worker}} - \underbrace{[(1-\phi)w(\theta_H;\pi) + \phi w(\theta_L;\pi)]}_{\text{Exp wage for unqualified worker}} \quad \text{(C21)}$$
$$= (2\phi - 1)[w(\theta_H;\pi) - w(\theta_L;\pi)].$$

explicitly as a function of π and parameters of the model (which are suppressed in the notation above). The symmetric equilibria in the examples below are computed by solving the equation $\pi = G(I(\pi))$ numerically.

One advantage with this parametrization is that *symmetric* equilibria can be shown to be unique under some restrictions on the cost distribution G.

C.1 Task Assignments for Symmetric Equilibria

We proceed by calculating conditions on parameters α, ϕ, π such that the solution to the firms' problem (C19) belongs to a given type.

POSSIBILITY 1: The solution to (C19) is $\sigma^* = 1$ and $\gamma^* = 0$ *if* $\alpha \le \phi$ *and* $\pi \le \frac{\alpha+\phi-1}{2\phi-1}$

If $\sigma^* = 1$ and $\gamma^* = 0$. then since the Kuhn-Tucker conditions are necessary and sufficient, this solves the problem if and only if

$$\frac{d}{d\sigma}C(1,0,\pi)^{\alpha}S(1,0,\pi)^{1-\alpha} \ge 0 \text{ and } \frac{d}{d\sigma}C(1,0,\pi)^{\alpha}S(1,0,\pi)^{1-\alpha} \le 0 \quad \text{(C22)}$$

Now,

$$\frac{d}{d\sigma}C(\sigma,\gamma,\pi)^{\alpha}S(\sigma,\rho,\pi)^{1-\alpha} \quad \text{(C23)}$$
$$= \alpha\left(\frac{S(\sigma,\rho,\pi)}{C(\sigma,\rho,\pi)}\right)^{1-\alpha}\frac{\partial C(\sigma,\rho,\pi)}{\partial\sigma} + (1-\alpha)\left(\frac{C(\sigma,\rho,\pi)}{S(\sigma,\rho,\pi)}\right)^{\alpha}\frac{\partial S(\sigma,\rho,\pi)}{\partial\sigma}$$
$$\frac{d}{d\gamma}C(\sigma,\gamma,\pi)^{\alpha}S(\sigma,\rho,\pi)^{1-\alpha}$$
$$= \alpha\left(\frac{S(\sigma,\rho,\pi)}{C(\sigma,\rho,\pi)}\right)^{1-\alpha}\frac{\partial C(\sigma,\rho,\pi)}{\partial\gamma} + (1-\alpha)\left(\frac{C(\sigma,\rho,\pi)}{S(\sigma,\rho,\pi)}\right)^{\alpha}\frac{\partial S(\sigma,\rho,\pi)}{\partial\gamma}$$

Evaluated at $\sigma = 1$ and $\gamma = 0$ we thus have that the necessary and sufficient conditions for a solution of this form is that

$$\begin{aligned}
0 &\leq \alpha\left(\frac{S(1,0,\pi)}{C(1,0,\pi)}\right)^{1-\alpha}\frac{\partial C(1,0,\pi)}{\partial\sigma}+(1-\alpha)\left(\frac{C(1,0,\pi)}{S(1,0,\pi)}\right)^{\alpha}\frac{\partial S(1,0,\pi)}{\partial\sigma}\Leftrightarrow \\
0 &\leq \alpha\left(\frac{S(1,0,\pi)}{C(1,0,\pi)}\right)\frac{\partial C(1,0,\pi)}{\partial\sigma}+(1-\alpha)\frac{\partial S(1,0,\pi)}{\partial\sigma}= \\
&= \alpha\left(\frac{(1-\phi)\pi+\phi(1-\pi)}{\phi\pi}\right)\phi\pi-(1-\alpha)(\phi\pi+(1-\phi)(1-\pi))= \\
&= \alpha((1-\phi)\pi+\phi(1-\pi))-(1-\alpha)(\phi\pi+(1-\phi)(1-\pi))= \\
&= \pi(\alpha(1-2\phi)+(1-\alpha)(1-2\phi))+\alpha\phi-(1-\alpha)(1-\phi)= \\
&= \pi(1-2\phi)-1+\alpha+\phi\Leftrightarrow\pi\leq\frac{\alpha+\phi-1}{2\phi-1}
\end{aligned} \tag{C24}$$

(which is the first condition), and that

$$\begin{aligned}
0 &\geq \alpha\left(\frac{S(1,0,\pi)}{C(1,0,\pi)}\right)^{1-\alpha}\frac{\partial C(1,0,\pi)}{\partial\gamma}+(1-\alpha)\left(\frac{C(1,0,\pi)}{S(1,0,\pi)}\right)^{\alpha}\frac{\partial S(1,0,\pi)}{\partial\gamma}\Leftrightarrow \\
0 &\geq \alpha\left(\frac{S(1,0,\pi)}{C(1,0,\pi)}\right)\frac{\partial C(1,0,\pi)}{\partial\gamma}+(1-\alpha)\frac{\partial S(1,0,\pi)}{\partial\gamma}= \\
&= \alpha\left(\frac{(1-\phi)\pi+\phi(1-\pi)}{\phi\pi}\right)(1-\phi)\pi-(1-\alpha)[(1-\phi)\pi+\phi(1-\pi)] \\
&= \frac{[(1-\phi)\pi+\phi(1-\pi)]}{\phi}(\alpha(1-\phi)-(1-\alpha)\phi)\Leftrightarrow\alpha\leq\phi
\end{aligned} \tag{C25}$$

POSSIBILITY 2: The solution to (C19) is $(\sigma^*,\gamma^*)=\left(1,\frac{\alpha-\phi}{1-\phi}\right)$ *if* $\alpha>\phi$.

Assuming $\sigma^*=1$ and $\gamma^*>0$ is an optimal solution implies

$$\frac{d}{d\sigma}C(1,\gamma^*,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha}\geq 0 \text{ and } \frac{d}{d\gamma}C(1,\gamma^*,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha}=0. \tag{C26}$$

Note that (C23) implies that $\frac{d}{d\gamma}C(1,\gamma^*,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha}=0\Rightarrow\frac{d}{d\sigma}C(1,\gamma^*,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha}>0$. The idea for this is simply that if the first condition holds, then expected productivity is equalized for low signals, which implies that high signal workers are more productive in the complex task since they are more likely to be qualified. Hence, we only need to check that we get a valid solution to the equality, that is

$$
\begin{aligned}
0 &= \frac{d}{d\gamma}C(1,\gamma^*,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha} \Leftrightarrow \qquad \text{(C27)}\\
0 &= \alpha\left(\frac{S(1,\gamma^*,\pi)}{C(1,\gamma^*,\pi)}\right)\frac{\partial C(1,\gamma^*,\pi)}{\partial\gamma}+(1-\alpha)\frac{\partial S(1,\gamma^*,\pi)}{\partial\gamma}\\
&= \alpha\left(\frac{(1-\gamma^*)\left[(1-\phi)\pi+\phi(1-\pi)\right]}{\phi\pi+\gamma^*(1-\phi)\pi}\right)(1-\phi)\pi-(1-\alpha)\left[(1-\phi)\pi+\phi(1-\pi)\right] \Leftrightarrow\\
0 &= \alpha\left(\frac{(1-\gamma^*)}{\phi\pi+\gamma^*(1-\phi)\pi}\right)(1-\phi)\pi-(1-\alpha) \Leftrightarrow\\
0 &= \alpha(1-\gamma^*)(1-\phi)-(1-\alpha)(\phi+\gamma^*(1-\phi)) = \alpha-\phi-\gamma^*(1-\phi) \Leftrightarrow \gamma^* = \frac{\alpha-\phi}{1-\phi}
\end{aligned}
$$

For this to be a valid solution $\gamma^* \in (0,1)$, which is the case whenever $\alpha > \phi$.

POSSIBILITY 3: The solution to (C19) is $(\sigma^*,\gamma^*) = \left(\frac{\alpha}{\phi\pi+(1-\phi)(1-\pi)},0\right)$ *if* $\alpha \le \phi$ *and* $\pi > \frac{\alpha+\phi-1}{2\phi-1}$

The final possibility is that the solution is $0 < \sigma^* < 1$ and $\gamma^* = 0$. Then

$$
\frac{d}{d\sigma}C(\sigma^*,0,\pi)^{\alpha}S(\sigma^*,0,\pi)^{1-\alpha} = 0 \text{ and } \frac{d}{d\gamma}C(\sigma^*,0,\pi)^{\alpha}S(\sigma^*,0,\pi)^{1-\alpha} \le 0 \qquad \text{(C28)}
$$

Again the inequality is implied by the equality. Hence we need to check that

$$
\begin{aligned}
0 &= \alpha\left(\frac{S(\sigma^*,0,\pi)}{C(\sigma^*,0,\pi)}\right)\frac{\partial C(\sigma^*,0,\pi)}{\partial\sigma}+(1-\alpha)\frac{\partial S(\sigma^*,0,\pi)}{\partial\sigma} \qquad \text{(C29)}\\
&= \alpha\left(\frac{(1-\sigma^*)\left[\phi\pi+(1-\phi)(1-\pi)\right]+(1-\phi)\pi+\phi(1-\pi)}{\sigma^*\phi\pi}\right)\phi\pi\\
&\quad -(1-\alpha)(\phi\pi+(1-\phi)(1-\pi)) \Leftrightarrow\\
0 &= (\alpha(1-\sigma^*)-(1-\alpha)\sigma^*)(\phi\pi+(1-\phi)(1-\pi))+\alpha((1-\phi)\pi+\phi(1-\pi))\\
&= (\alpha-\sigma^*)(\phi\pi+(1-\phi)(1-\pi))+\alpha((1-\phi)\pi+\phi(1-\pi)) \Leftrightarrow\\
\sigma^* &= \frac{\alpha}{\phi\pi+(1-\phi)(1-\pi)}
\end{aligned}
$$

Since $\sigma^* > 0$ all that remains to be checked is that $\sigma^* < 1$, which is when

$$
\begin{aligned}
1 &> \frac{\alpha}{\phi\pi+(1-\phi)(1-\pi)} \Leftrightarrow \phi\pi+(1-\phi)(1-\pi) > \alpha \qquad \text{(C30)}\\
&\Leftrightarrow \quad \pi > \frac{\alpha+\phi-1}{2\phi-1}.
\end{aligned}
$$

This can only hold if $\alpha < \phi$

The three cases are mutually exclusive. Moreover, no other type of solution can exist since there must always be some workers with the high (low) signal in the complex (simple) task and since mixing both types can't be a solution. Taken together the first and third possibility constitutes what is referred to as "Case 1" in the subsections below with $\alpha \leq \phi$ and the second possibility corresponds to "Case 2".

C.2 Reduced Form Expressions For Benefits to Invest

C.2.1 Case 1: $\alpha \leq \phi$

Using the equilibrium characterization in terms of expected marginal products we have that if $\pi > \frac{\alpha+\phi-1}{2\phi-1}$, since workers with the high signal are assigned to the simple task, workers with high and low signals earn the same wage and incentives are zero. If $\pi > \frac{\alpha+\phi-1}{2\phi-1}$ on the other hand the relevant marginal products are

$$\begin{aligned} w(\theta_H;\pi) &= \frac{\phi\pi}{\phi\pi+(1-\phi)(1-\pi)}\alpha\left(\frac{(1-\phi)\pi+\phi(1-\pi)}{\phi\pi}\right)^{1-\alpha} \\ w(\theta_L;\pi) &= (1-\alpha)\left(\frac{\phi\pi}{(1-\phi)\pi+\phi(1-\pi)}\right)^{\alpha}. \end{aligned} \tag{C31}$$

The incentives to invest are thus

$$\begin{aligned} I(\pi) &= (2\phi-1)(w(\theta_H;\pi)-w(\theta_L;\pi)) \\ &= (2\phi-1)\frac{\phi\pi}{\phi\pi+(1-\phi)(1-\pi)}\alpha\left(\frac{(1-\phi)\pi+\phi(1-\pi)}{\phi\pi}\right)^{1-\alpha} \\ &\quad -(2\phi-1)(1-\alpha)\left(\frac{\phi\pi}{(1-\phi)\pi+\phi(1-\pi)}\right)^{\alpha} \\ &= (2\phi-1)\left(\frac{\phi\pi}{(1-\phi)\pi+\phi(1-\pi)}\right)^{\alpha}\left(\alpha\frac{(1-\phi)\pi+\phi(1-\pi)}{\phi\pi+(1-\phi)(1-\pi)}-(1-\alpha)\right) = \\ &= (2\phi-1)\left(\frac{\phi\pi}{(1-\phi)\pi+\phi(1-\pi)}\right)^{\alpha}\left(\frac{\alpha-\phi\pi+(1-\phi)(1-\pi)}{\phi\pi+(1-\phi)(1-\pi)}\right) \\ &= k\left(\frac{\phi\pi}{\phi-k\pi}\right)^{\alpha}\left(\frac{\alpha}{k\pi+(1-\phi)}-1\right), \end{aligned} \tag{C32}$$

where $k \equiv 2\phi-1$. Now,

$$\begin{aligned} 0 &\leq k\left(\frac{\phi\pi}{\phi-k\pi}\right)^{\alpha}\left(\frac{\alpha}{k\pi+(1-\phi)}-1\right) \Rightarrow \\ 0 &\leq \frac{\alpha}{k\pi+(1-\phi)}-1 \Leftrightarrow k\pi+(1-\phi)\leq\alpha \Leftrightarrow \pi\leq\frac{\alpha+\phi-1}{k}=\frac{\alpha+\phi-1}{2\phi-1}, \end{aligned} \tag{C33}$$

so

$$I(\pi) = \max\left\{k\left(\frac{\phi\pi}{\phi-k\pi}\right)^{\alpha}\left(\frac{\alpha}{k\pi+(1-\phi)}-1\right),0\right\} \tag{C34}$$

C.2.2 Case 2: $\alpha > \phi$

The wages are now

$$w(\theta_H;\pi) = \frac{\phi\pi}{\phi\pi + (1-\phi)(1-\pi)}\alpha\left(\frac{S(1,\gamma^*,\pi)}{C(1,\gamma^*,\pi)}\right)^{1-\alpha} \tag{C35}$$
$$w(\theta_L;\pi) = \frac{(1-\phi)\pi}{(1-\phi)\pi + \phi(1-\pi)}\alpha\left(\frac{S(1,\gamma^*,\pi)}{C(1,\gamma^*,\pi)}\right)^{1-\alpha}$$

And, using the closed form expression $\gamma^* = \frac{\alpha-\phi}{1-\phi}$ we have that

$$\frac{S(1,\gamma^*,\pi)}{C(1,\gamma^*,\pi)} = \frac{(1-\gamma^*)\left[(1-\phi)\pi + \phi(1-\pi)\right]}{\phi\pi + \gamma^*(1-\phi)\pi} = \frac{(1-\alpha)\left((1-\phi)\pi + \phi(1-\pi)\right)}{\pi\alpha(1-\phi)}, \tag{C36}$$

so

$$\begin{aligned}
I(\pi) &= k\left(\frac{\phi\pi}{\phi\pi + (1-\phi)(1-\pi)} - \frac{(1-\phi)\pi}{(1-\phi)\pi + \phi(1-\pi)}\right)\alpha\left(\frac{(1-\alpha)\left((1-\phi)\pi + \phi(1-\pi)\right)}{\pi\alpha(1-\phi)}\right)^{1-\alpha} \\
&= k\left(\frac{\phi\left((1-\phi)\pi + \phi(1-\pi)\right)}{\left((\phi\pi + (1-\phi)(1-\pi))\right)(1-\phi)} - 1\right)(1-\alpha)\left(\frac{\pi\alpha(1-\phi)}{(1-\alpha)\left((1-\phi)\pi + \phi(1-\pi)\right)}\right)^{\alpha} \\
&= k\left(\frac{\phi - (\phi\pi + (1-\phi)(1-\pi))}{(\phi\pi + (1-\phi)(1-\pi))(1-\phi)}\right)(1-\alpha)\left(\frac{\pi\alpha(1-\phi)}{(1-\alpha)\left((1-\phi)\pi + \phi(1-\pi)\right)}\right)^{\alpha} \\
&= k\left(\frac{\phi}{(\phi\pi + (1-\phi)(1-\pi))} - 1\right)\frac{(1-\alpha)(1-\phi)^{\alpha}\alpha^{\alpha}}{(1-\phi)(1-\alpha^{a})}\left(\frac{\pi}{((1-\phi)\pi + \phi(1-\pi))}\right)^{\alpha} \\
&= \frac{k\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(1-\phi)^{1-\alpha}}\left(\frac{\phi}{k\pi + (1-\phi)} - 1\right)\left(\frac{\pi}{\phi - k\pi}\right)^{\alpha} \\
&= M\left(\frac{\phi}{k\pi + (1-\phi)} - 1\right)\left(\frac{\pi}{\phi - k\pi}\right)^{\alpha},
\end{aligned} \tag{C37}$$

C.3 Uniqueness of Symmetric Equilibria

Proposition 1 *Suppose that G is concave and that $\underline{c} < 0$. Then there is a unique symmetric equilibrium π^* (which is non-trivial).*

This result is useful because it makes comparisons between situations with and without discrimination more straightforward. With multiplicity of symmetric equilibria we would have to either make set-wise comparisons or to select a plausible symmetric equilibrium according to some criterion.

Proof. Case1: $\alpha \leq \phi$

In this case the optimal solution to the task assignment problem (C19) is to set $(\sigma(\pi), \gamma(\pi)) = (1, 0)$ when $0 \leq \pi \leq \frac{\alpha+\phi-1}{2\phi-1}$. The associated incentives (see C34 above) are

$$I(\pi) = \max\left\{k\left(\frac{\phi\pi}{\phi - k\pi}\right)^{\alpha}\left(\frac{\alpha}{k\pi + (1-\phi)} - 1\right), 0\right\} \quad \text{for } k \equiv 2\phi - 1,$$

where the reason for the max-operator is that $\frac{\alpha}{k\pi+(1-p)}-1<0$ when $\pi>\frac{\alpha+p-1}{2p-1}$, which is the range where $\sigma(\pi)>0$ and incentives consequently are equal to zero. Define

$$J(\pi)=\left(\frac{\pi}{\phi-k\pi}\right)^{\alpha}\left(\frac{\alpha}{k\pi+(1-\phi)}-1\right), \tag{C38}$$

so that $I(\pi)$ in (C34) is given by $I(\pi)=k\phi^{\alpha}J(\pi)$ whenever $I(\pi)>0$.

STEP 1: G UNIFORM: If G is uniform $G(I(\pi))=QJ(\pi)+R$ for any $I(\pi)\in[\underline{c},\overline{c}]$, where $Q=\frac{1}{[\underline{c},\overline{c}]}k\phi^{\alpha}$ and $R=-\frac{\underline{c}}{[\underline{c},\overline{c}]}>0$ ($\underline{c}<0$ by assumption). By a direct calculation we have that

$$J'(\pi)=J(\pi)\,\alpha\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)(\alpha-k\pi-(1-\phi.))}\right). \tag{C39}$$

A sufficient condition for uniqueness is that $\frac{d}{d\pi}G(I(\pi^*))<1$ in any equilibrium π^* (since $\underline{c}<0\Rightarrow G(I(0))$ is above the diagonal). We drop the $*-$superscript for equilibria and note that an equilibrium point satisfies $\pi=G(I(\pi))=QJ(\pi)+R$, so

$$\begin{aligned}\frac{d}{d\pi}G(I(\pi)) &= QJ'(\pi)=QJ(\pi)\,\alpha\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)^2(\alpha-k\pi-(1-\phi.))}\right) \\ &= (\pi-R)\,\alpha\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)(\alpha-k\pi-(1-\phi.))}\right) \\ &< \pi\alpha\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)(\alpha-k\pi-(1-\phi.))}\right),\end{aligned} \tag{C40}$$

where the equality is from evaluating the derivative at an equilibrium and the inequality follows since $R>0$. The expression on the third line of (C40) is increasing in α and $\alpha\leq\phi$, so

$$\begin{aligned}\frac{d}{d\pi}G(I(\pi)) &< \pi\alpha\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)(\alpha-k\pi-(1-\phi.))}\right) \\ &\leq \pi\phi\left(\frac{\phi}{\pi(\phi-k\pi)}-\frac{k}{(k\pi+1-\phi)(\phi-k\pi-(1-\phi.))}\right) \\ &= \frac{\phi^2}{\phi-k\pi}-\frac{\pi\phi}{(k\pi+1-\phi)(1-\pi)}.\end{aligned} \tag{C41}$$

Now, $k\pi+1-\phi<k+(1-\phi)=\phi\Rightarrow\frac{\phi}{k\pi+1-\phi}>1$, implying that

$$\frac{d}{d\pi}G(I(\pi))<\frac{\phi^2}{\phi-k\pi}-\frac{\pi}{(1-\pi)}=\frac{\phi^2}{\phi-(2\phi-1)\pi}-\frac{\pi}{(1-\pi)}. \tag{C42}$$

Differentiating and simplifying we find that $\frac{d}{d\phi}\left(\frac{\phi^2}{\phi-(2\phi-1)\pi}\right)>0$ and since $\phi\in\left[\frac{1}{2},1\right]$ the derivative $\frac{d}{d\pi}G(I(\pi))$ is bounded by the right hand side of (C42) evaluated at $\phi=1$. That is, $\frac{d}{d\pi}G(I(\pi))<\frac{1}{1-\pi}-\frac{\pi}{(1-\pi)}=1$, which establishes that equilibria must be unique when G is uniform.

STEP 2: G CONCAVE: For a general concave distribution we note that for every $c \in [\underline{c}, \overline{c}]$, the mean value theorem implies that there exists $c^* \in [\underline{c}, c]$ such that $G(c) - G(0) = G(c) = G'(c^*)\, c$, hence the equilibrium must satisfy $\pi = G(I(\pi)) = G'(c^*)\, I(\pi)$ for some $c^* \leq I(\pi)$. Concavity implies that $G'(I(\pi)) \leq G'(c^*)$, so

$$\frac{d}{d\pi} G(I(\pi)) = G'(I(\pi))\, I'(\pi) \leq G'(c^*)\, I'(\pi) = G'(c^*)\, k\phi^{\alpha} J'(\pi) = QJ'(\pi) \tag{C43}$$

for $Q = G'(c^*)\, k\phi^{\alpha}$. At this point it is just to proceed as with a uniform distribution (with $R = 0$).

Case 2: $\alpha > \phi$

This case can be handled in a similar way. We have shown above (see C37) that the incentives in this case are given by

$$I(\pi) = M\left(\frac{\phi}{k\pi + (1-\phi)} - 1\right)\left(\frac{\pi}{\phi - k\pi}\right)^{\alpha}$$

where $M = \frac{k\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(1-\phi)^{1-\alpha}}$. Compared with the previous case there are only two changes. The first is that the constant has changed, which doesn't matter since the constant disappears when evaluating the derivative at an equilibrium. The second change is that the term $\frac{\phi}{k\pi+(1-\phi)}$ replaces $\frac{\alpha}{k\pi+(1-\phi)}$, which does affect the remaining calculations.

Again assuming a uniform distribution and differentiating and evaluating at an equilibrium point as in the case with $\alpha \leq \phi$ now gives

$$\begin{aligned} \frac{d}{d\pi} G(I(\pi)) &= (\pi - R)\,\alpha\left(\frac{\phi}{\pi(\phi - k\pi)} - \frac{1}{(k\pi + 1 - \phi)(1-\pi)}\right) \\ &< \alpha\pi\left(\frac{\phi}{\pi(\phi - k\pi)} - \frac{1}{(k\pi + 1 - \phi)(1-\pi)}\right) \\ &< \frac{\phi}{(\phi - k\pi)} - \frac{\pi}{(k\pi + 1 - \phi)(1-\pi)} \end{aligned} \tag{C44}$$

Observe that

$$\begin{aligned} \frac{d}{d\phi}\left[\frac{\phi}{(\phi - k\pi)}\right] &= \frac{d}{d\phi}\left[\frac{\phi}{(\phi - (2\phi - 1)\pi)}\right] = \frac{(\phi - (2\phi-1)\pi) - \phi(1 - 2\pi)}{(\phi - (2\phi - 1)\pi)^2} \\ &= \frac{-(2\phi - 1)\pi + 2\phi\pi}{(\phi - (2\phi - 1)\pi)^2} = \frac{\pi}{(\phi - (2\phi - 1)\pi)^2} > 0. \end{aligned} \tag{C45}$$

Hence,

$$\begin{aligned}
\frac{d}{d\pi}G\left(I\left(\pi\right)\right) &< \frac{\phi}{(\phi-k\pi)}-\frac{\pi}{(k\pi+1-\phi)\left(1-\pi\right)}<\frac{1}{1-\pi}-\frac{\pi}{(k\pi+1-\phi)\left(1-\pi\right)} \qquad \text{(C46)}\\
&= \frac{1}{1-\pi}\left(1-\frac{\pi}{(k\pi+1-\phi)}\right)=\frac{1}{1-\pi}\left(\frac{k\pi+1-\phi-\pi}{k\pi+1-\phi}\right)\\
&= \frac{1}{1-\pi}\left(\frac{(2\phi-1)\,\pi+1-\phi-\pi}{k\pi+1-\phi}\right)=\frac{1}{1-\pi}\left(\frac{(1-\phi)\left(1-2\pi\right)}{k\pi+1-\phi}\right)\\
&= \frac{1-2\pi}{1-\pi}\frac{(1-\phi)}{k\pi+1-\phi}<1,
\end{aligned}$$

which proves the claim.

C.4 Equilibria with Full Segregation

Given that we *impose* that all members of group B are assigned to the simple task we have that the factor inputs are

$$\begin{aligned}
C(\sigma,\gamma,\pi) &= \lambda\left(\sigma\phi\pi+\gamma(1-\phi)\pi\right) \qquad \text{(C47)}\\
S(\sigma,\gamma,\pi) &= \lambda\left((1-\sigma)\left[\phi\pi+(1-\phi)\left(1-\pi\right)\right]+(1-\gamma)\left[(1-\phi)\pi+\phi\left(1-\pi\right)\right]\right)+(1-\lambda)\,,
\end{aligned}$$

where π now is the fraction of investors in group W (π^W in the main document) and all other variables really should have the $W-$superscript as well. However, the model now collapses to something which is almost the model with a single group. Indeed define

$$\Lambda=\frac{1-\lambda}{\lambda}$$

and everything is as if there is some exogenous extra input of S given by Λ. Our goal is to find conditions on ϕ,Λ,α,π such that the solution to the firms' problem belongs to a given type.

Case 1 *The solution to the task assignment problem is* $(\sigma^*,\lambda^*)=(1,0)$ *if*

1. $\left[(1-\phi)\pi+\phi\left(1-\pi\right)\right]\left(\alpha-\phi\right)+\alpha\Lambda\left(1-\phi\right)\leq 0$ *and*

2. $\pi\leq\frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$

Guessing that $(\sigma, \lambda) = (1, 0)$ solves the task assignment problem and proceeding as in the single group model we find that then

$$
\begin{aligned}
0 &\leq \alpha \left(\frac{(1-\phi)\pi + \phi(1-\pi) + \Lambda}{\phi\pi} \right) \phi\pi - (1-\alpha)(\phi\pi + (1-\phi)(1-\pi)) \Leftrightarrow \\
0 &\leq \alpha((1-\phi)\pi + \phi(1-\pi) + \Lambda) - (1-\alpha)(\phi\pi + (1-\phi)(1-\pi)) \\
&= \pi(\alpha(1-2\phi) + (1-\alpha)(1-2\phi)) + \alpha(\phi + \Lambda) - (1-\alpha)(1-\phi) \\
&= \pi(1-2\phi) + \alpha\Lambda + \alpha + \phi - 1 \\
\pi &\leq \frac{\alpha(1+\Lambda) + \phi - 1}{2\phi - 1},
\end{aligned}
$$

for the optimality condition for $\sigma = 1$ to hold. Next the condition for $\gamma = 0$ becomes

$$
\begin{aligned}
0 &\geq \alpha \left(\frac{(1-\phi)\pi + \phi(1-\pi) + \Lambda}{\phi\pi} \right) (1-\phi)\pi - (1-\alpha)[(1-\phi)\pi + \phi(1-\pi)] \\
&= ((1-\phi)\pi + \phi(1-\pi)) \left(\frac{(1-\phi)\alpha}{\phi} - (1-\alpha) \right) + \frac{\alpha\Lambda(1-\phi)}{\phi} \\
&= ((1-\phi)\pi + \phi(1-\pi)) \left(\frac{\alpha - \phi}{\phi} \right) + \frac{\alpha\Lambda(1-\phi)}{\phi} \Leftrightarrow \\
0 &\geq ((1-\phi)\pi + \phi(1-\pi))(\alpha - \phi) + \alpha\Lambda(1-\phi)
\end{aligned}
$$

Case 2 *he solution to the task assignment problem is* $(\sigma^*, \gamma^*) = \left(1, \frac{\alpha-\phi}{1-\phi} + \frac{\alpha\Lambda}{(1-\phi)\pi + \phi(1-\pi)}\right)$ *if*

1. $[(1-\phi)\pi + \phi(1-\pi)](\alpha - \phi) + \alpha\Lambda(1-\phi) > 0$ *and*

2. $\pi < \frac{(1-\alpha)\phi - \alpha\Lambda(1-\phi)}{(2\phi-1)(1-\alpha)}$

The second possibility is that $\sigma^* = 1$ and $\gamma^* > 0$ in an optimal solution. For the same reasons as in the symmetric equilibrium case we only need to check there is some $\gamma^* \in (0, 1)$ solving

$$
\begin{aligned}
0 &= \frac{d}{d\gamma} C(1, \gamma^*, \pi)^\alpha S(1, \gamma^*, \pi)^{1-\alpha} \Leftrightarrow \\
0 &= \alpha \left(\frac{S(1, \gamma^*, \pi)}{C(1, \gamma^*, \pi)} \right) \frac{\partial C(1, \gamma^*, \pi)}{\partial\gamma} + (1-\alpha) \frac{\partial S(1, \gamma^*, \pi)}{\partial\gamma} \\
&= \alpha \left(\frac{(1-\gamma^*)[(1-\phi)\pi + \phi(1-\pi)] + \Lambda}{(\phi\pi + \gamma^*(1-\phi)\pi)} \right) (1-\phi)\pi - (1-\alpha)[(1-\phi)\pi + \phi(1-\pi)] \Leftrightarrow
\end{aligned}
$$

$$\begin{aligned}
0 &= [(1-\phi)\pi + \phi(1-\pi)]\left(\frac{\alpha(1-\gamma^*)(1-\phi)}{\phi+\gamma^*(1-\phi)} - (1-\alpha)\right) + \alpha\frac{\Lambda(1-\phi)}{\phi+\gamma^*(1-\phi)} \Leftrightarrow \\
&= [(1-\phi)\pi + \phi(1-\pi)]\left(\frac{\alpha(1-\gamma^*)(1-\phi) - (1-\alpha)(\phi+\gamma^*(1-\phi))}{\phi+\gamma^*(1-\phi)}\right) + \alpha\frac{\Lambda(1-\phi)}{\phi+\gamma^*(1-\phi)} \\
&= [(1-\phi)\pi + \phi(1-\pi)]\left(\frac{\alpha-\phi-\gamma^*(1-\phi)}{\phi+\gamma^*(1-\phi)}\right) + \alpha\frac{\Lambda(1-\phi)}{\phi+\gamma^*(1-\phi)} \Leftrightarrow \\
0 &= [(1-\phi)\pi + \phi(1-\pi)](\alpha-\phi-\gamma^*(1-\phi)) + \alpha\Lambda(1-\phi) \Leftrightarrow \\
\gamma^* &= \frac{\alpha-\phi}{1-\phi} + \frac{\alpha\Lambda}{(1-\phi)\pi+\phi(1-\pi)}
\end{aligned}$$

So $\gamma^* > 0$ requires that $((1-\phi)\pi + \phi(1-\pi))(\alpha-\phi) + \alpha\Lambda(1-\phi) > 0$ and $\gamma^* < 1$ that

$$\begin{aligned}
1 &> \frac{\alpha-\phi}{1-\phi} + \frac{\alpha\Lambda}{(1-\phi)\pi+\phi(1-\pi)} \Leftrightarrow \\
\frac{1-\alpha}{1-\phi} &> \frac{\alpha\Lambda}{(1-\phi)\pi+\phi(1-\pi)} 0 \Leftrightarrow \\
(1-\alpha)[(1-\phi)\pi+\phi(1-\pi)] &> \alpha\Lambda(1-\phi) \Leftrightarrow \\
\pi &< \frac{(1-\alpha)\phi - \alpha\Lambda(1-\phi)}{(2\phi-1)(1-\alpha)}
\end{aligned}$$

Case 3 *The solution to the task assignment problem is* $(\sigma^*, \gamma^*) = \left(\frac{\alpha(1+\Lambda)}{\phi\pi+(1-\phi)(1-\pi)}, 0\right)$ *if* $\pi > \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$

Suppose that $\sigma^* < 1$ and $\gamma^* = 0$ solves problem. Again, we can argue as in the symmetric case to show that it is sufficient to find a $\sigma^* \in (0,1)$ solving

$$\begin{aligned}
0 &= \alpha\left(\frac{S(\sigma^*,0,\pi)}{C(\sigma^*,0,\pi)}\right)\frac{\partial C(\sigma^*,0,\pi)}{\partial\sigma} + (1-\alpha)\frac{\partial S(\sigma^*,0,\pi)}{\partial\sigma} &\text{(C48)} \\
&= \alpha\left(\frac{(1-\sigma^*)[\phi\pi+(1-\phi)(1-\pi)] + (1-\phi)\pi+\phi(1-\pi)+\Lambda}{\sigma^*\phi\pi}\right)\phi\pi \\
&\quad -(1-\alpha)(\phi\pi+(1-\phi)(1-\pi)) \Leftrightarrow \\
0 &= (\phi\pi+(1-\phi)(1-\pi))(\alpha(1-\sigma^*) - \sigma^*(1-\alpha)) + \alpha((1-\phi)\pi+\phi(1-\pi)+\Lambda) \\
&= (\phi\pi+(1-\phi)(1-\pi))(\alpha-\sigma^*) + \alpha((1-\phi)\pi+\phi(1-\pi)+\Lambda) &\text{(C49)} \\
&= (\alpha-\sigma^*)(\phi\pi+(1-\phi)(1-\pi)) + \alpha((1-\phi)\pi+\phi(1-\pi)+\Lambda) \Leftrightarrow \\
\sigma^* &= \frac{\alpha(1+\Lambda)}{\phi\pi+(1-\phi)(1-\pi)},
\end{aligned}$$

Clearly $\sigma^* > 0$, so the question is whether $\sigma^* < 1$, that is if

$$\phi\pi + (1-\phi)(1-\pi) > \alpha(1+\Lambda) \Leftrightarrow \pi > \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1} \tag{C50}$$

Case 4 *The solution to the task assignment problem is* $(\sigma^*,\gamma^*) = (1,1)$ *if* $.\pi \geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(1-\alpha)(2\phi-1)}$

Final possibility. $(\sigma^*,\gamma^*) = (1,1)$, which requires that

$$\begin{aligned} 0 &\leq \frac{d}{d\gamma}C(1,1,\pi)^{\alpha}S(1,\gamma^*,\pi)^{1-\alpha} \Leftrightarrow \quad \text{(C51)} \\ 0 &\leq \alpha\left(\frac{S(1,1,\pi)}{C(1,1,\pi)}\right)\frac{\partial C(1,1,\pi)}{\partial\gamma} + (1-\alpha)\frac{\partial S(1,1,\pi)}{\partial\gamma} \\ &= \alpha\left(\frac{\Lambda}{\pi}\right)(1-\phi)\,\pi - (1-\alpha)\left[(1-\phi)\pi + \phi\,(1-\pi)\right] \\ &= \alpha\Lambda\,(1-\phi) - (1-\alpha)\left[(1-\phi)\pi + \phi\,(1-\pi)\right] \\ (1-\alpha)\,(1-2\phi)\pi &\leq \alpha\Lambda\,(1-\phi) - (1-\alpha)\,\phi \Leftrightarrow \quad \text{(C52)} \end{aligned}$$

C.4.1 Summary

To check consistency of the four cases this is what we got out.

Condition	Solution
$\pi > \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$	$(\sigma^*,\gamma^*) = \left(\frac{\alpha(1+\Lambda)}{(\phi\pi+(1-\phi)(1-\pi))}, 0\right)$
$\pi \leq \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$ $0 \geq ((1-\phi)\pi + \phi\,(1-\pi))\,(\alpha-\phi) + \alpha\Lambda\,(1-\phi)$	$(\sigma^*,\gamma^*) = (1,0)$
$\pi < \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(2\phi-1)(1-\alpha)}$ $0 < ((1-\phi)\pi + \phi\,(1-\pi))\,(\alpha-\phi) + \alpha\Lambda\,(1-\phi)$	$(\sigma^*,\gamma^*) = \left(1, \frac{\alpha-\phi}{1-\phi} + \frac{\alpha\Lambda}{(1-\phi)\pi+\phi(1-\pi)}\right)$
$\pi \geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(1-\alpha)(2\phi-1)}$	$(\sigma^*,\gamma^*) = (1,1)$

1. If $\alpha \geq \phi$. Then

- $0 \geq ((1-\phi)\pi + \phi\,(1-\pi))\,(\alpha-\phi) + \alpha\Lambda\,(1-\phi)$ can never hold

- $0 < ((1-\phi)\pi + \phi\,(1-\pi))\,(\alpha-\phi) + \alpha\Lambda\,(1-\phi)$ holds for all $\pi > 0$

- $\frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1} > \frac{\phi(1+\Lambda)+\phi-1}{2\phi-1} = 1 + \frac{\phi\Lambda}{2\phi-1}$, so $\pi > \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$ can never hold for any $\pi \leq 1$

2. If $\Lambda \geq \frac{(1-\alpha)\phi}{\alpha(1-\phi)}$. Then $\pi \geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(1-\alpha)(2\phi-1)}$ always holds
3. If $\Lambda \leq \frac{1-\alpha}{\alpha}$. Then

$$\begin{aligned} \frac{(1-\alpha)\,\phi - \alpha\Lambda\,(1-\phi)}{(1-\alpha)\,(2\phi-1)} &\geq \frac{(1-\alpha)\,\phi - \alpha\frac{1-\alpha}{\alpha}\,(1-\phi)}{(1-\alpha)\,(2\phi-1)} \\ &= \frac{\phi-(1-\phi)}{(2\phi-1)} = 1, \end{aligned}$$

so $\pi \geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(1-\alpha)(2\phi-1)}$ can never hold for any $\pi \in [0,1]$

4. Hence, suppose that

$$\begin{aligned} \pi &> \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1} \\ \pi &\geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(1-\alpha)(2\phi-1)} \end{aligned}$$

would hold simultaneously for some $\pi \in [0,1]$. For the first equation to be satisfied it must be that

$$\begin{aligned} \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1} &\leq 1 \\ \frac{\phi-\alpha}{\alpha} &\geq \Lambda \end{aligned}$$

and for the second equation to hold it must be that $\Lambda > \frac{1-\alpha}{\alpha}$ which is a contradiction since we get

$$\frac{\phi-\alpha}{\alpha} < \frac{1-\alpha}{\alpha} < \Lambda \leq \frac{\phi-\alpha}{\alpha}.$$

Hence we can make the following summary:

If $a \geq \phi$, then the following parameter ranges are the relevant:

$0 \leq \Lambda \leq \frac{1-\alpha}{\alpha}$	$\frac{1-\alpha}{\alpha} \leq \Lambda \leq \frac{(1-\alpha)\phi}{\alpha(1-\phi)}$	$\frac{(1-\alpha)\phi}{\alpha(1-\phi)} \leq \Lambda$
mix b	mix b if $\pi < \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(2\phi-1)(1-\alpha)}$ All in high if $\pi \geq \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(2\phi-1)(1-\alpha)}$	All in high task

If $a < \phi$, then we have the following

$0 < \Lambda \leq \frac{\phi-\alpha}{\alpha}$	$\frac{\phi-\alpha}{\alpha} \leq \Lambda \leq \frac{(\phi-\alpha)\phi}{\alpha(1-\phi)}$	$\frac{(1-\alpha)\phi}{\alpha(1-\phi)} \leq \Lambda$
ATS if $\pi \leq \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$ mix g if $\pi > \frac{\alpha(1+\Lambda)+\phi-1}{2\phi-1}$	ATS if $\pi \leq \frac{(\phi-\alpha)\phi-\alpha\Lambda(1-\phi)}{(\phi-\alpha)(2\phi-1)} \in [0,1]$ Mix b if $\frac{(\phi-\alpha)\phi-\alpha\Lambda(1-\phi)}{(\phi-\alpha)(2\phi-1)} < \pi < \frac{(1-\alpha)\phi-\alpha\Lambda(1-\phi)}{(\phi-\alpha)(2\phi-1)}$	All in high task

D Technology and Incentives to Segregate

The parameter α also matters for the incentives to discriminate. With some stretching, one may think of an increase in α as "skill-biased technical change". It then seems natural to ask whether such technical change will increase or decrease the incentives to discriminate. Potentially, this could provide a technological explanation for Civil Rights legislation and the abolishment of slavery or apartheid. The reader should take notice that *some* aspects of the patterns displayed in this

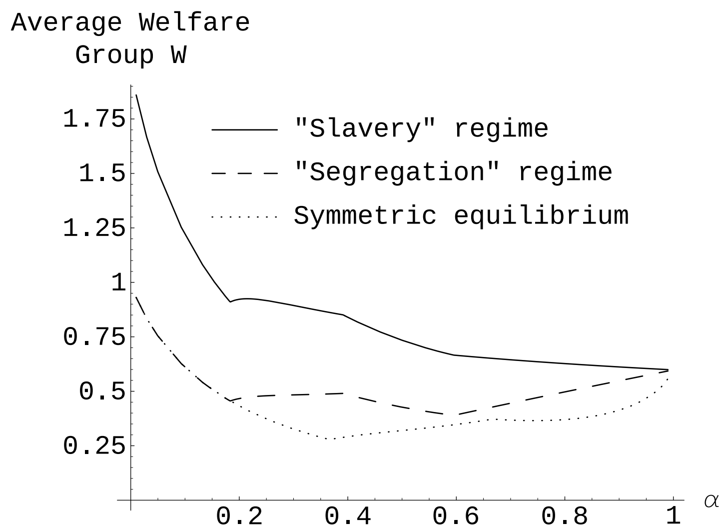

Figure D1: Welfare of the dominant group under three different regimes as a function of α

example are parameter-specific, even within this very restricted parametric class, so this section is only meant to be suggestive.

We have let the technology parameter α vary and kept all other parameters as in the example in Table 1 in the main text. Figure D1 compares the average welfare of the dominant group in three different regimes. The dotted line correspond to the unique, symmetric equilibrium. The dashed line refers to the "segregation regime" in which all B workers are forced *by law* to be employed in the simple task. Finally, the continuous line in the figure plots the welfare of the dominant group in a regime where, in addition to being segregated, the B workers' earnings are expropriated by the W workers (we label this the "slavery" regime). The "segregation" and "slavery" regimes correspond to the same labor market outcomes and differ only in the distribution of property rights over wages.

Changes in α affect the possibility to support segregation in equilibrium in a straightforward manner. Segregation is harder to sustain in equilibrium when α increases. In the example the threshold is approximately at $\alpha = 0.76$, and for lower α segregation is an equilibrium also without mandated segregation, whereas the force of law is necessary for higher values of α.

The effects on *incentives* to segregate are more subtle. When α is small, most workers are employed in the simple task in either regime. In particular, a fraction of W workers with high signals are employed in the simple task in both regimes, implying that aggregate inputs are identical in all regimes.[D1] There are therefore no gains from segregation for W workers, whereas slavery is still advantageous because they expropriate group B.

When α is high, forcing the segregated group into the simple task has negligible effects on output and productivity for workers in the complex task. Segregation then becomes like a scaled down version of the symmetric equilibrium, where the labor of the segregated group is almost totally wasted. The gains for W workers are small, and the losses for society as a whole are large. Since there is little to steal from B workers, slavery is also unprofitable.

The big gains from segregation occur for intermediate values of α, when segregating group B into the simple task has significant effects on the productivity of W workers. However, as is evident from Figure D1, the payoff difference between segregation and the color-blind equilibrium is not necessarily single-peaked. There are two peaks in the payoff difference in the example, which has to do qualitative changes in the type of equilibrium as α increases. Some W workers are in the simple task if α is low. This implies that the wage for W workers with signal θ_L decreases as α increases because of a decline the marginal productivity in the complex task. Eventually, all W workers are assigned to the complex task, and at this point (which is around $\alpha = 0.6$) the wage of the low signal workers start to increase in α (see Figure D2).

Harder to understand is the behavior of the wage for W workers with signal θ_H. After being increasing in α for a while, it begins to decrease at $\alpha = 0.4$. Again, this corresponds with a qualitative change in the equilibrium. The peak at $\alpha = 0.4$ in Figure D2 corresponds with a change from only signal θ_H workers being assigned to the complex task to a situation where a fraction of the θ_L workers are also assigned to the complex task. At this point the factor ratio starts to increase at a fast enough rate so that incentives are affected negatively and π^W begins to decrease in α, up to the point where the economy runs out of W workers to reassign to the complex task (at $\alpha = 0.6$, where the high signal wage starts to increase again).

The discussion above suggests that the two local maxima with respect to the payoff differ-

[D1] Labor market outcomes coincide for small values of α because a fraction of W workers with signal θ_H are employed in the simple task. There are therefore no incentives to invest and the equilibrium investment is $\pi^W = G(0)$ both with and without segregation. The factor ratio is the same in all regimes, there is no effect on the marginal productivity in the complex task, and therefore no incentive to segregate.

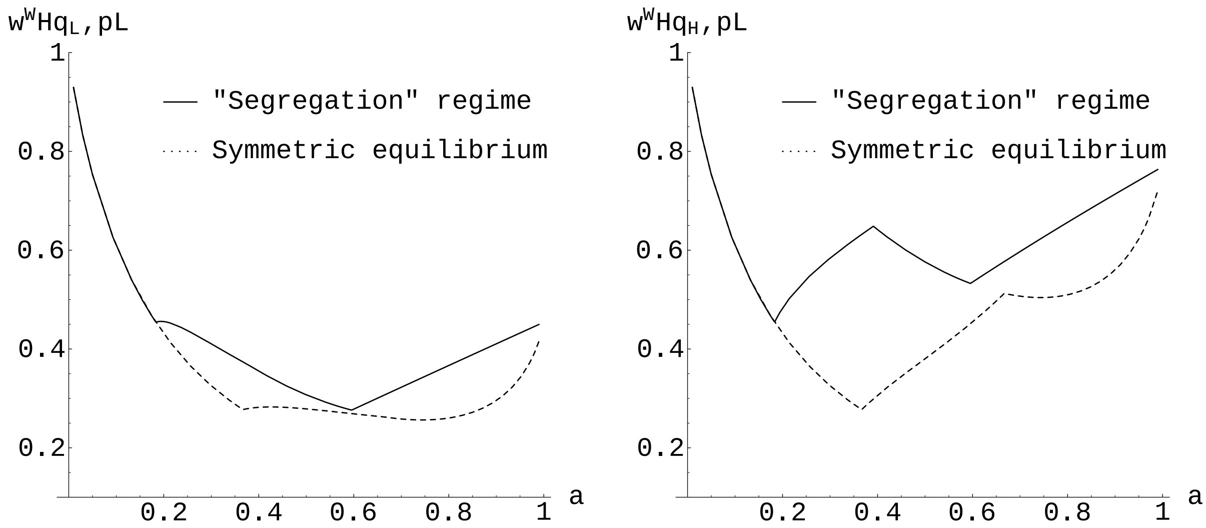

Figure D2: Wage under two different regimes as a function of α

ence between the segregation regime and the symmetric equilibrium may be an artefact of the parametrization. In spite of the many different effects that are active, it is possible that a richer set of signals would create a more systematic pattern, but this would be computationally more demanding and we have not tried this yet.

In sum, what seems robust is that incentives to segregate are small for very low and very high α. In an intermediate range the gains are substantial, but behaves non-systematically both within and across examples.