

# Endogenous Comparative Advantage\*

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August 4, 2017

## Abstract

We develop a stylized model of trade between identical countries. The departure from neoclassical theory is that endogenous human capital investments are imperfectly observed. Investments have a public good component: firms use aggregate country investment as the prior when evaluating workers, which creates an informational externality. The interaction between this externality and general equilibrium effects creates asymmetric equilibria with comparative advantages even when there is a unique equilibrium under autarky. Symmetric, no-trade equilibria may be unstable under free trade. Welfare effects are ambiguous: trade may be Pareto improving even if it leads to an asymmetric equilibrium with rich and poor countries.

**Keywords:** Comparative Advantage, Specialization, Human Capital.

**JEL Classification Number:** D62, D82, F11, O12

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\*It is nearly impossible to thank everybody that contributed comments and suggestions. We thank them all. Special thanks to Lutz Hendricks, Pat Kehoe, Tim Kehoe, Narayana Kocherlakota, John Knowles, Rody Manuelli, Andrew Postlewaite, Paul Segerstrom, Ananth Seshadri, Robert Staiger, Kjetil Storesletten, Scott Taylor and Fabrizio Zilibotti for helpful comments, suggestions and discussions. We also thank the editor and two anonymous referees for their comments. Support from NSF Grants #SES-0003520 and #SES-0001717 is gratefully acknowledged. The research for this paper was done partly when Peter Norman was visiting IIES Stockholm and he is grateful for their hospitality.

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# 1 Introduction

In this paper we develop a stylized model of international trade in which the interaction between general equilibrium effects and an informational externality generates a novel explanation for specialization and trade. Competitive firms in two *ex ante identical* countries have access to the same technology. Two goods are produced using two types of labor input: *skilled* and *unskilled* labor. A “high tech” good can be produced from skilled labor only, whereas skilled and unskilled labor are perfect substitutes in the production of a “low-tech” good.<sup>1</sup> Trade is frictionless, but there is an informational friction in the labor market.

Two crucial assumptions drive our results: 1) workers acquire their skills from costly human capital investments, and 2) firms observe a noisy measure of skills. These two assumptions generate an informational externality because the wage distribution in a country depends on beliefs about human capital investments, which in a rational expectations equilibrium are determined by average human capital investments. This implies that investments in human capital have a “public good component”: higher average human capital investments increase expected wages for all workers, regardless of their human capital.

Conditional on human capital investments in the two countries, the model is like a Heckscher-Ohlin model, except that individual wages in the high-tech sector are higher for workers that are more likely to be skilled. Firms estimate workers’ productivity by Bayesian updating, using a noisy signal of skill and a prior that is determined by average investment in the country, which in our setup is the equilibrium proportion of skilled workers.

In equilibrium, workers drawing a high signal are paid more than workers who draw a low signal, because they are more likely to be skilled. This generates incentives to invest in human capital. In addition, the wage schedule is affected by the relative goods price. If the high-tech good becomes more expensive, individual incentives to acquire human capital increase, because wages of workers in the high-tech sector increase. Workers have different investment costs, but the cost distribution is the same across countries. The condition that determines the equilibrium proportion of investors is that a worker invests if the cost is below the expected wage increase.

The interaction between price effects and the informational externality creates a force in favor of specialization that, to our knowledge, is novel in the international trade literature. Equilibria emerge where countries specialize, endogenously, despite being fundamentally identical *ex ante*, that is, before human capital investment takes place. In these equilibria, workers from the country specializing in the production of the high-tech good on average enjoy higher wages, but spend more resources on human capital investment.

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<sup>1</sup>In a previous version we considered a more general technology with one good being more intensive in skilled labor than the other. This generalization creates no additional qualitative insights.

To understand how asymmetric equilibria can emerge in our model, assume that the skill level in the foreign country increases. This has two effects in the foreign country. An immediate effect is that individual workers are more likely to be skilled, which may improve the incentives for skill acquisition.<sup>2</sup> In addition, the human capital increase changes the production possibilities set so that the equilibrium production of the high-tech good increases, reducing its relative price, thereby reducing the wage differential between skilled and unskilled workers. This second effect, a pecuniary externality, affects both countries, but the first direct effect only occurs in the country where the skill level is changed. Hence, wages are affected asymmetrically, which makes it possible that incentives for human capital investments go in opposite directions in the two countries.

The informational asymmetry is essential to generate endogenous specialization. The effects on relative prices from increasing the relative abundance of skilled labor is the same as in a perfect information model. In a perfect information analogue of our model factor price equalization implies that incentives to acquire skills are independent of location, a property which falls apart when skills are observed with noise.

We show parameterizations of the model where asymmetric equilibria arise under free trade. There is always at least one symmetric equilibrium with no gainful trade that replicates the autarky allocation. Equilibrium multiplicity is a natural feature of these models, but trade and specialization do not arise simply from countries coordinating on different equilibria of a model with multiple autarky equilibria.<sup>3</sup> We show that asymmetric equilibria occur even if the autarky equilibrium is unique. However, the stability conditions under autarky differ from the stability conditions under free trade. We find that opening up international trade may destabilize a unique and stable autarky equilibrium. Hence in our model cross country income differences may be an inevitable aspect of free trade.

Rich countries are better off than poor countries, but this does not necessarily imply that the poor country is worse off under trade than in autarky. Welfare in the poor country can go either way, but we emphasize the less intuitive result showing an example where an asymmetric equilibrium Pareto-dominates the autarky equilibrium. The intuition is that an increase in the skill level abroad may drive down the relative price of the high-tech good so much that exchanging the low-tech good for the high-tech good generates higher welfare in the poor country compared to domestic production.

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<sup>2</sup>The relationship between aggregate skills and incentives for skill acquisition is non-monotonic. In the extreme case in which all workers have high skills firms ignore the noisy signal and pay all workers the value of the marginal product of a skilled worker, which completely eliminates any incentive to invest in human capital. At the other extreme, when nobody has high skills, all workers are employed in the low-tech sector and are paid the marginal product of an unskilled worker, again with zero incentives to acquire skills.

<sup>3</sup>Matsuyama (2002) argues why multiplicity by itself does not offer a compelling reason for observed asymmetries.

There is no systematic advantage for large economies in our model. Equilibria of the two-country model can be reinterpreted as an equilibrium of a  $n$  country extension, where some countries specialize in the high-tech industry. Only the aggregate size of the part of the world economy that specializes in the high-tech sector matters. The model is thus consistent with a world where there is no particular relationship between size and development.

To what extent is the mechanism highlighted by the model empirically relevant? To answer this question one must first address whether the assumptions driving the results are reasonable. The labor economics literature has highlighted that informational frictions of the type we assume in our model significantly affect workers' wages.<sup>4</sup> Farber and Gibbons (1996) and Altonji and Pierret (2001) first showed that employers' learning is significant, providing support to the assumption that employers initially observe workers' skills with noise.<sup>5</sup> Recent literature confirms these results.<sup>6</sup> In particular, Lange (2007) measured the "speed" of employer learning finding that, according to the best estimates, it takes three years for an employer to reduce her expectation error to 50 percent of its initial value, and 26 years to reduce it to less than 10 percent of its initial value. Despite the relatively fast initial learning, considering that average employer tenure is currently just above 4 years on average,<sup>7</sup> these figures support a significant scope for the mechanism proposed in this paper to play a role in determining workers' wage distribution and incentives.

Based on this evidence, we cannot dismiss the possibility that these type of informational asymmetries may also play a role in explaining, at least in part, trade and specialization across countries. A full empirical investigation of the model implications, which would require accounting for (and separately identifying) other relevant factors, is outside the scope of this paper.

The rest of the paper is structured as follows. The next section discusses the contribution of this paper relative to existing literature. Section 3 introduces the model, defines the equilibrium, and shows that it can be characterized as a planner's problem, simplifying the following analysis. Section 4 characterizes in detail the equilibria under autarky. The main result, the existence of equilibria with trade and specialization, is presented in Section 5.

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<sup>4</sup>In most of the literature, the identification of the main effect exploits panel data where workers are observed over time. If employers imperfectly observe workers' skills, but learn over time through the observation of productivity signals, then as tenure increases wages should become more correlated with measures of productivity available to the researcher (typically, workers' scores in aptitude tests).

<sup>5</sup>However, according to Altonji and Pierret (2001) interpretation of the results, the mechanism does not explain racial differences, as suggested by the statistical discrimination literature.

<sup>6</sup>See e.g. Schönberg (2007), Pinkston (2009), and Kahn and Lange (2014) using U.S. data, and Lesner (2017) with Danish data. Cornwell et al. (2017) use Brazilian data to show that employer variation in workers' perceived race significantly affects wages, which is consistent with a model of statistical discrimination.

<sup>7</sup>4.2 years in January 2016 according to the U.S. Bureau of Labor Statistics news release "Employee Tenure", <https://www.bls.gov/news.release/tenure.toc.htm>, last accessed: August 2, 2017.

Section 6 discusses the stability and welfare properties of equilibria with trade. Section 7 concludes discussing the robustness of the results to extending the model to multiple countries, to including physical capital in the production, and migration.

## 2 Related literature

Our main contribution, relative to the literature, is to suggest a novel source of trade and comparative advantage between *identical* countries. There are several papers in the literature that include some of the crucial elements of our model, endogenous and imperfectly observed human capital accumulation, but in those models either some exogenous differences are posited, or equilibrium multiplicity is the driving source of specialization. One should also note that our results are robust to introducing exogenous productivity differences. If one country has a “fundamental” comparative advantage in the high-tech industry, it may still specialize as a low-tech industry as a result of the mechanism in our model, provided that the exogenous differences are small enough.

Our model belongs to the literature on trade and endogenous skill formation initiated by [Findlay and Kierzkowski \(1983\)](#), who develop a general equilibrium model where the driver of trade is endogenous human capital acquisition. The factors of production are skilled and unskilled labor as in our model, but countries specialize because of exogenous differences in the availability of inputs needed to acquire human capital, generating what we refer to as price effects. In our setup instead, countries are identical also in the cost of acquiring human capital.<sup>8</sup>

Among the papers presenting models with asymmetric information, [Grossman and Maggi \(2000\)](#) and [Grossman \(2004\)](#) have elements that are similar to our setup: a Heckscher-Ohlin model with imperfectly observable skills. Their focus is on showing the effect of skill distributions. For their purposes it is sufficient to consider how trade is affected by exogenous differences in the talent distribution across countries, therefore they ignore the incentives to acquire skills that are central in our model.<sup>9</sup>

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<sup>8</sup>The focus of this literature is mainly in showing how even if factor price equalization holds (for the marginal worker), trade induces different incentives to acquire human capital across countries. For recent extensions see also [Ranjan \(2001\)](#), [Falvey et al. \(2010\)](#), [Auer \(2015\)](#), [Unel \(2015\)](#), [Blanchard and Willmann \(2016\)](#), and [Danziger \(2017\)](#). In some cases, the exogenous country differences are assumed by analyzing the effects of trade on a small open economy that takes the world price as given, as in [Cartiglia \(1997\)](#), [Bougheas et al. \(2011\)](#), [Bonfatti and Ghatak \(2013\)](#), or [Harris and Robertson \(2013\)](#).

<sup>9</sup>A number of papers study the effects various informational asymmetries on trade. Their focus is essentially on analyzing the effects of asymmetric information and not on studying how trade arises in equilibrium. [Vogel \(2007\)](#), focuses on the effect of institutional quality reducing workers’ moral hazard. [Davidson and Sly \(2014\)](#), focus on how opening to trade affects one country’s distortions in human capital accumulations when education has a signaling role. [Park \(2011\)](#) analyzes trade agreements under imperfect public monitoring, [Zhang \(2012\)](#) consider effects of asymmetric information when exporters are credit constrained, and

Costinot (2009), like us, seeks to formulate a more fundamental theory of comparative advantage. The technology is also based on the idea that human capital is more important for some firms than for others. The main difference is that the model ultimately derives cross country differences from exogenous differences in institutional quality and human capital.

Chisik (2003) derives trade in a model where products may acquire, in equilibrium, different reputation for quality. Self-fulfilling reputation determines the average quality of a country exports, and comparative advantages arise endogenously because countries coordinate on selecting different equilibria.<sup>10</sup> Similarly, in Chatterjee (2015) comparative advantages emerge endogenously as a Nash equilibrium of a game in which countries choose policies that affect sector-specific productivities or relative factor endowments. In these papers equilibrium multiplicity is needed to generate the comparative advantage. In our model instead, trade may arise even when there is a unique autarky equilibrium.

While our underlying assumptions are very different, our model shares many features with trade models with increasing returns (Ethier (1982), Krugman (1980)), their versions usually referred to as “agglomeration models” (Krugman and Venables (1995), Puga and Venables (1999)), and the “symmetry-breaking” literature (see Matsuyama (1996, 2004)). Agglomeration models can sustain a concentration of (high-income) manufacturing because production costs decrease with the size of the industry. Manufactured goods are inputs in the production of other goods, implying that being close to other producers saves on transportation costs. This creates incentives to concentrate production. When production costs are neither too small nor too large, there are equilibria where manufacturing is concentrated in one country, that becomes richer.

While our model is considerably less complicated and closer to the neoclassical benchmark than models with increasing returns, there is a close similarity in how a pecuniary externality interacts with local market conditions. However, there are also crucial differences: our model resorts to imperfect information rather than global increasing returns, and to decentralized decisions instead of a planner’s choice of technology. Agglomeration models predict a positive relation between size and development whereas our model has no such implications, as illustrated in Section 6.5. Moreover, we show that the specialization into rich and poor country may in our model Pareto dominate autarky. We are not aware

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Creane and Jeitschko (2016) show that weak institutions may result in welfare-reducing trade in an adverse selection model. Razin and Sadka (2003) use an informational asymmetry to model the role of foreign direct investments, Casella and Rauch (2002) derive a role for minority groups in international trade using an informational friction, and McCalman (2002) considers the impact of asymmetric information in bargaining about trade agreements. Eicher (1999) considers a model that is significantly richer than ours in many ways, but the informational asymmetry is modeled in reduced form.

<sup>10</sup>Other models based on trust and endogenous quality reputation are Araujo and Ornelas (2007), Rasmusen (2017), and Basu and Chau (1998), who assume countries are initially asymmetric as they differ in the endowment of human capital.

of any increasing returns model with this feature.

We borrow some of the modeling assumptions from the statistical discrimination literature. In [Moro and Norman \(2004\)](#) racial differences arise in a statistical discrimination model because groups specialize in the level of acquired skill. Here, countries take the role of racial groups, but we extend the model to the production of two goods to allow for trade. This complicates the analysis, therefore we simplify the model on some non-crucial features: the information technology only allows for two types of signal, the production technology is linear, and complementarities arise here because of Cobb-Douglas preferences.

### 3 The Model

Two countries, labeled by  $j = h, f$ , are populated by a continuum of agents, where  $\lambda^h$  and  $\lambda^f = 1 - \lambda^h$  denote the mass of agents in each country. Agents are price takers. We build on a simple  $2 \times 2 \times 2$  trade model but with factors of production being workers with and without human capital. The model is closed by a stylized model of human capital acquisition and an informational technology borrowed from the statistical discrimination literature.<sup>11</sup> Workers cannot migrate.

All agents can invest in human capital by paying an investment cost  $c$  drawn from a cumulative density  $G$  defined on the interval  $[\underline{c}, \bar{c}]$ . Investment is binary, the investment cost  $c$  is private information and is independent of which country the agent lives in. We call workers who invest in human capital *qualified* and the others *unqualified*. Agents have the same preferences. The utility of an agent consuming the bundle  $(x_1, x_2)$  is  $u(x_1, x_2) - c$  if the agent invests and  $u(x_1, x_2)$  otherwise, where  $u$  is a homothetic and strictly quasi-concave.

After the investments, nature assigns each worker a signal  $\theta \in \{g, b\}$  that employers observe. For simplicity we assume that

$$\Pr [g|\text{worker qualified}] = \Pr [b|\text{worker unqualified}] = \eta > \frac{1}{2}, \quad (1)$$

where the restriction that  $\eta > 1/2$  labels signals so that  $g$  is good news. Our preferred interpretation is that the unobservable investment is a costly effort decision and the signal is an imperfect measure of the costly effort, aggregating information from letter of recommendation, grades, tests, etc. . . .

The two consumption goods are produced solely from qualified and unqualified labor, denoted  $q$  and  $n$  respectively, according to

$$y_1(q, n) = q; \quad y_2(q, n) = q + n. \quad (2)$$

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<sup>11</sup>See [Coate and Loury \(1993\)](#)

All workers are thus perfect substitutes in industry 2, whereas only qualified workers contribute to the production of good 1.<sup>12</sup>

After defining equilibrium, we show in Subsection 3.2 that given human capital investment the equilibrium in the goods and labor markets can be characterized as the solution to a planners' problem, simplifying the derivations that follow. Section 3.3 shows how technology can be represented graphically by a production possibilities set.

### 3.1 Equilibrium

Our notion of equilibrium is analogous to a competitive equilibrium in a perfect information environment, but the informational asymmetry makes the treatment of the "labor supply" somewhat non-standard: skilled labor is endogenously determined by incentives that depend on prices derived from the goods markets.

Consider an agent with realized wage  $w$  deciding how to allocate her earnings between the two goods given prices  $p = (p_1, p_2)$ . The (ex-post) maximized utility of the worker is

$$v(w, p) = \max_{x_1, x_2} u(x_1, x_2) \quad (3)$$

subject to  $p_1 x_1 + p_2 x_2 \leq w$ .

By strict quasi-concavity of  $u(x_1, x_2)$ , the optimization problem in (3) has a unique solution, and, with the usual notational abuse, we denote the demand functions by  $x_1(w, p), x_2(w, p)$ .

Employers cannot observe if a worker is qualified, so a labor demand is a map  $l : \{g, b\} \rightarrow R_+$ . Denote with  $\pi$  any fraction of qualified workers in a country. This fraction can be thought of as the *prior* probability of a worker being qualified, before employers observes the signal. Employers then use Bayes' rule to form the posterior probability that a worker is qualified given her signal:

$$\mu(g, \pi) \equiv \frac{\eta \pi}{\eta \pi + (1 - \eta)(1 - \pi)} \quad \mu(b, \pi) \equiv \frac{(1 - \eta) \pi}{(1 - \eta) \pi + \eta(1 - \pi)}. \quad (4)$$

Associated with any fraction of qualified workers,  $\pi$ , and a given labor demand  $l$ , the corresponding quantities of qualified and unqualified workers are:

$$\begin{aligned} q &= l(g) \mu(g, \pi) + l(b) \mu(b, \pi) \\ n &= l(g) (1 - \mu(g, \pi)) + l(b) (1 - \mu(b, \pi)), \end{aligned} \quad (5)$$

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<sup>12</sup>This extreme technology is for simplicity only. Qualitatively, we need two sectors with different factor intensities, just like in the Hecksher-Ohlin model with fixed factor endowments.



We assume that a strong law of large numbers applies and treat  $q$  and  $n$  in (5) as both expected and realized inputs of labor.

Without loss of generality there is a representative firm in each sector and each country, which takes a *wage schedule*  $w^j : \{g, b\} \rightarrow R_+$  and output prices  $p_i$  as given.<sup>13</sup> Using the production function (2) and (5), the profit maximization problem for a Sector 1 firm is

$$\max_l p_1 (l(g) \mu(g, \pi^j) + l(b) \mu(b, \pi^j)) - w_g^j l(g) - w_b^j l(b), \quad (6)$$

For Sector 2, where qualified and unqualified workers are equally productive, the profit maximization problem is

$$\max_l p_2 (l(g) + l(b)) - w_g^j l(g) - w_b^j l(b). \quad (7)$$

Agents have rational expectations about the wages and prices, but face uncertainty about the realization of the signal. Denoting  $v(w, p)$  the indirect utility function defined in (3), the expected utility for an agent in country  $j$  with investment cost  $c$  is

$$\eta v(w_g^j, p) + (1 - \eta) v(w_b^j, p) - c \quad (8)$$

if a worker invest in human capital, and

$$(1 - \eta) v(w_g^j, p) + \eta v(w_b^j, p) \quad (9)$$

if not. The worker is better off investing if and only if (8) exceeds (9), or if the cost of investment is less than the gross incentives:  $c \leq (2\eta - 1) \cdot (v(w_g^j, p) - v(w_b^j, p))$ . The implied proportion of investors in country  $j$  is thus

$$\pi^j = G \left( (2\eta - 1) (v(w_g^j, p) - v(w_b^j, p)) \right). \quad (10)$$

To sum up: optimal consumption plans are defined in (3), (6) and (7) describe the profit maximization problems for each sector, and (10) summarizes the individually optimal human capital investments.

What remains to describe are the market clearing conditions. Factor market clearing requires that the aggregate demand for workers with each signal equals the mass of agents who draw the signal. That is, let  $l_i^j = (l_i^j(g), l_i^j(b))$  be a labor demand scheme in industry

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<sup>13</sup>The caveat is that the informational asymmetry would disappear if (qualified) workers could start their own firms. We rule this and other contractual solutions to the informational asymmetry out by assumption. One way to justify this is to assume that there is a minimum efficient scale for production and that only aggregate output, and not the performance of individual workers, can be observed.

$i$  and country  $j$  and write the labor market clearing conditions as

$$\begin{aligned} l_1^j(g) + l_2^j(g) &= \eta\pi^j + (1-\eta)(1-\pi^j) \\ l_1^j(b) + l_2^j(b) &= (1-\eta)\pi^j + \eta(1-\pi^j). \end{aligned} \quad (11)$$

Finally, for the product market equilibrium conditions it is convenient to let  $x_i^j$  be the output in industry  $i$  and country  $j$ . That is

$$\begin{aligned} x_1^j &= l_1^j(g)\mu(g, \pi^j) + l_1^j(b)\mu(b, \pi^j) \\ x_2^j &= l_2^j(g) + l_2^j(b), \end{aligned} \quad (12)$$

which allows us to write the product market clearing conditions for the world market as

$$\sum_{j=h,f} \lambda^j \left( x_i^j - \underbrace{[\eta\pi^j + (1-\eta)(1-\pi^j)]}_{\text{\#agents with wage } w_g^j} x_i(w_g^j, p) - \underbrace{[(1-\eta)\pi^j + \eta(1-\pi^j)]}_{\text{\#agents with wage } w_b^j} x_i(w_b^j, p) \right) = 0 \quad (13)$$

Our definition of equilibrium is then:

**Definition 1** *A competitive equilibrium consists of output prices  $p^*$ , wages  $w^{j*}$ , labor demands  $l_i^{j*}$ , outputs  $x_i^{j*}$ , and fractions of qualified workers  $\pi^{j*}$  for each country  $j = h, f$  and industry  $i = 1, 2$ , satisfying:*

(a)  $l_1^{j*}$  solves (6) and  $l_2^{j*}$  solves (7) given  $p_i = p_i^*$  and  $x_1^{j*}$  and  $x_2^{j*}$  are the associated profit maximizing outputs in  $j = h, f$

(b) the product market clearing conditions in (13) are satisfied.

(c) the factor market clearing conditions in (11) are satisfied.

(d)  $\pi^{j*}$  satisfies (10) given  $p = p^*$  and wages  $w^j = w^{j*}$  for  $j = h, f$

We refer to a situation where all equilibrium conditions except the optimal investment condition (d) are fulfilled as a *continuation equilibrium*.<sup>14</sup>

### 3.2 A Planning Characterization of Continuation Equilibria

From the point of view of an informationally unconstrained planner, a continuation equilibrium is inefficient: qualified and unqualified workers with the same signal are treated symmetrically, resulting in a misallocation of workers to jobs. However, if the symmetric treatment of workers with the same signal is viewed as a fundamental property of the

<sup>14</sup>This term is mainly due to lack of a better alternative. Due to the workers being non-atomic it does not make a difference whether investments are made before or simultaneously with the wage posting.

environment, then the equilibrium allocation is (constrained) efficient *conditional on the investment behavior*. This allows us to describe aggregate equilibrium allocations as solutions to the planning problem:

$$\max_{(x_1, x_2) \in X^W(\pi^h, \pi^f)} u(x_1, x_2), \quad (14)$$

where  $X^W(\pi^h, \pi^f)$  is the world production possibilities set defined in Section 3.3.

The following proposition shows that, for *fixed* investments, versions of the welfare theorems hold: the equilibrium is characterized by a planning problem where the informational asymmetry is built into the feasible set. This allows us to appeal to simple graphs in the analysis that follows.

**Proposition 1** *Suppose that  $u(x_1, x_1)$  is homothetic. Then:*

1. *The aggregate world consumption in any continuation equilibrium is a solution to (14)*
2. *Suppose that  $(x_1^*, x_2^*)$  solves (14),  $(p_1^*, p_2^*)$  is a normal to a hyperplane that separates the set of bundles such that  $u(x_1, x_2) \geq u(x_1^*, x_2^*)$  and  $X^W(\pi^h, \pi^f)$ , and that  $w_g^{j*} = \max\{p_1^* \mu(g, \pi^j), p_2^*\}$  and  $w_b^{j*} = \max\{p_1^* \mu(b, \pi^j), p_2^*\}$  in each country  $j$ . Then these prices, wages and aggregate consumption are part of a continuation equilibrium.<sup>15</sup>*

Proposition 1 immediately implies that, given any  $(\pi^h, \pi^f)$  there is a unique continuation equilibrium up to a re-normalization of the prices.

### 3.3 The Production Possibilities Set

A useful way to represent technology is in terms of the *production possibilities set*. The set of feasible production plans in a country depends on the fraction of workers that invest in human capital,  $\pi$ . Figure 1 illustrates the (per capita) production possibilities set in a country, which we denote with  $X(\pi)$ .

To understand the figure, first observe that  $(x_1, x_2) = (0, 1)$  if all workers are producing good 2, and that  $(x_1, x_2) = (\pi, 0)$  if all workers are producing good 1, because only a fraction  $\pi$  of the workers are productive in Sector 1. There are  $\pi\eta + (1 - \pi)(1 - \eta)$  workers with signal  $g$  and  $\pi(1 - \eta) + (1 - \pi)\eta$  workers with signal  $b$ . If all signal  $g$  workers are in Sector 1 ( $\pi\eta$  of these workers are productive) and all signal- $b$  workers are in Sector 2, then the outputs are given by the point at the kink in the graph. The frontier to the right of the kink is steeper because in that region all  $g$  workers are employed in Sector 1, therefore to increase production firms must employ more  $b$  workers, who are less likely to be qualified. To the left of the kink instead, only  $g$  workers are employed in Sector 1.

<sup>15</sup>The allocation of workers in each country is somewhat complicated to describe in general, but is implicitly pinned down as the (almost always) unique worker allocation that can produce the equilibrium bundle.

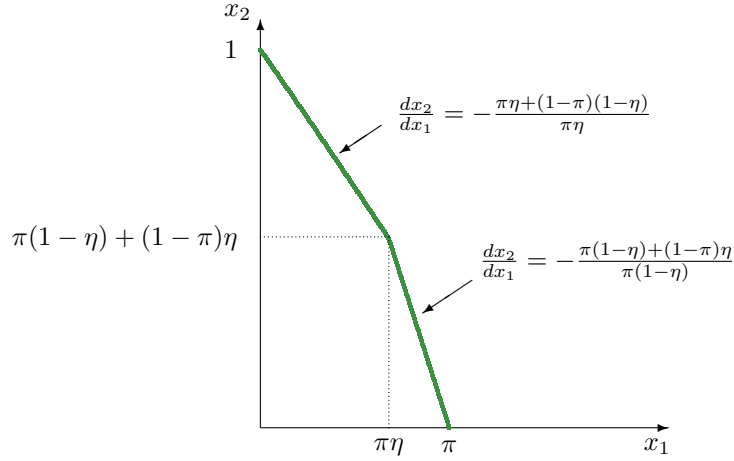


Figure 1: Per capita production possibilities in a country

The *world production possibilities* set is given by  $X^W(\pi^h, \pi^f) = \lambda^h X(\pi^h) + \lambda^f X(\pi^f)$  and is convex by convexity of  $X(\pi)$ . The next proposition immediately follows, since the production possibilities set becomes (weakly) flatter as investment in any country increases:

**Proposition 2** *Suppose that  $u(x_1, x_2)$  is homothetic. Then in any continuation equilibrium the relative price of the high-tech good is (weakly) decreasing in the countries' investment  $\pi^h$  and  $\pi^f$ .*

## 4 A Parametric Specification

While the results presented below are more general, for simplicity of exposition in the remainder of the paper we will restrict attention to Cobb-Douglas preferences,  $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ , which imply demand functions:

$$x_1(p, w) = \frac{\alpha w}{p_1} \quad x_2(p, w) = \frac{(1-\alpha)w}{p_2}. \quad (15)$$

The continuation utility for a worker that earns wage  $w$  is therefore:

$$v(w, p) = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{p_1^\alpha p_2^{1-\alpha}} w. \quad (16)$$

We normalize setting  $p_2 = 1$  and, with abuse of notation, write  $p(\boldsymbol{\pi})$ ,  $w_g^j(\boldsymbol{\pi})$  and  $w_b^j(\boldsymbol{\pi})$  for the unique continuation equilibrium prices and wages in good 2 units, where  $\boldsymbol{\pi} = (\pi^h, \pi^f)$ .

A qualified worker earns  $w_g^j(\boldsymbol{\pi})$  with probability  $\eta$  and  $w_b^j(\boldsymbol{\pi})$  with probability  $1-\eta$ . Symmetrically, an unqualified worker earns  $w_g^j(\boldsymbol{\pi})$  with probability  $1-\eta$  and  $w_b^j(\boldsymbol{\pi})$  with

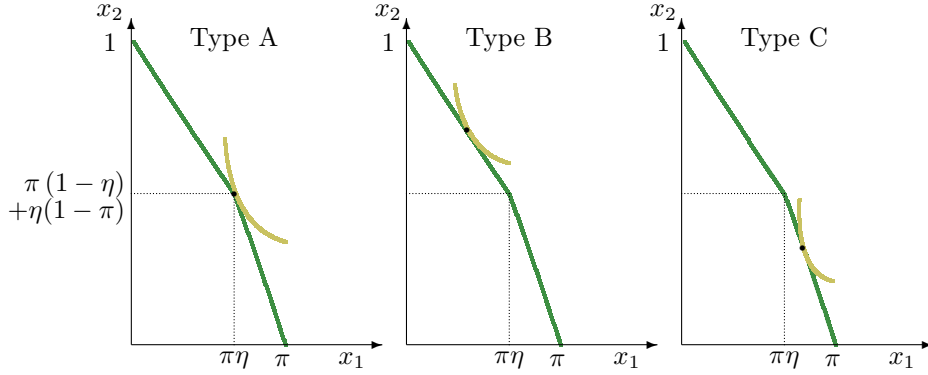


Figure 2: Three types of continuation equilibria

probability  $\eta$ . Computing the expectation of  $v(w, p)$  in (16) *conditional on investment* and subtracting from this the expectation of  $v(w, p)$  *conditional on not investing* we get the *gross benefits of investment* for an agent in country  $j$ , denoted  $B^j(\pi)$ , which is given by

$$\begin{aligned} B^j(\pi) &= E\{v(w, p) | \text{qualified}\} - E\{v(w, p) | \text{unqualified}\} \\ &= \alpha^\alpha (1 - \alpha)^{1 - \alpha} (2\eta - 1) \frac{(w_g^j(\pi) - w_b^j(\pi))}{(p(\pi))^\alpha}. \end{aligned} \quad (17)$$

Using condition (d) in Definition 1 we see that any  $\pi$  such that  $\pi^j = G(B^j(\pi))$  for  $j = h, f$  gives an equilibrium fraction of investors in each country. All that remains to calculate full equilibria is to derive expressions for the continuation equilibrium prices.

#### 4.1 Continuation Equilibria in Autarky

As a benchmark, we first consider a closed economy. Suppressing the country index, we write  $\pi$  for the proportion of qualified workers. There are three possible types of continuation equilibria, illustrated in Figure 2.<sup>16</sup>

**Type A equilibria (allocation of workers “according to signals”).** Graphically, this type occurs when the tangency is at the kink of the feasible set. That is, all workers with signal  $b$  ( $g$ ) are working in the low (high) tech sector. Outputs are then  $x_1 = \eta\pi$  and  $x_2 = (1 - \eta)\pi + \eta(1 - \pi)$ , so the demands in (15) pin down the price of the high-tech good

<sup>16</sup>This is a somewhat unfortunate aspect of having only 2 signals. With a continuum of signals we would get a strictly convex production possibilities set and the tangency condition would determine a unique threshold signal. However, as it is much simpler to compute explicit examples with two signals we decided to stick with the more inelegant case. Calculations are straightforward but may be tedious. We provided more detailed steps in the web appendix [Moro and Norman \(2017\)](#).

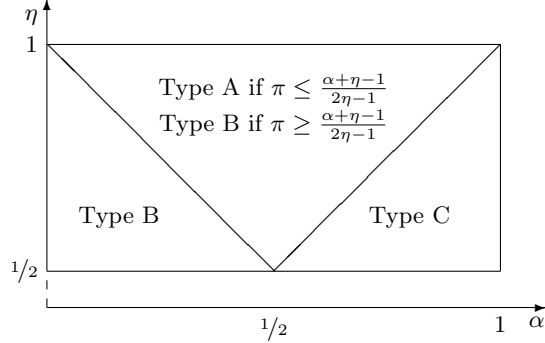


Figure 3: Types of autarky equilibria in the  $(\alpha, \eta)$  space

as

$$p(\pi) = \frac{\alpha}{1-\alpha} \frac{(1-\eta)\pi + \eta(1-\pi)}{\eta\pi}. \quad (18)$$

Candidate equilibrium wages are obtained by the zero profits condition. Since  $p_2 = 1$ , this immediately gives  $w_b(\pi) = 1$ . The high-tech firm sells  $\eta\pi$  units at price  $p(\pi)$  and hires  $\eta\pi + (1-\eta)(1-\pi)$  workers with signal  $g$ . Zero profits in Sector 1 therefore implies that the wage in that sector,  $w_g(\pi)$ , equals the price of good 1 times the expected probability that a worker with signal  $g$  is productive in that sector  $\mu(g, \pi)$ :

$$w_g(\pi) = p(\pi) \mu(g, \pi) = p(\pi) \frac{\pi\eta}{\pi\eta + (1-\eta)(1-\pi)}, \quad (19)$$

which has the alternative interpretation that the wage equals the expected value of output. Finally, we have to check that a high-tech firm has no incentive to hire signal  $b$  workers, and that a low-tech firm has no incentive to hire signal  $g$  workers. These conditions give rise to inequalities that determine the region where a Type A equilibrium exists (see Figure 3).

**Type B equilibria (mixing of good signals).** In Figure 2, this corresponds to a tangency to the left of the kink. Some workers with signal  $g$  work in Sector 2. These workers earn the same wage as  $g$ -signal workers in Sector 1, and, since all workers in the low-tech sector are paid their marginal product, 1, it follows immediately that  $w_g(\pi) = w_b(\pi) = 1$ . This provides workers zero incentives to invest. Because this makes the case less interesting for the full equilibrium of the model we refer the reader to the web appendix for details.

**Type C equilibria (mixing of bad signals).** This equilibrium occurs when the demand for the high-tech good is strong (i.e. when the Cobb-Douglas share of good 1 parameter  $\alpha$  is high). In Figure 2, this corresponds to a tangency to the right of the kink. In this case a fraction  $\beta$  of workers with signal  $b$  (defined below) works in Sector 1. Workers with signal

$b$  employed in the low-tech sector are paid 1. Signal- $b$  workers in the high tech sector must be paid their expected productivity, which equals the price times their probability of being productive, or  $p(\pi) \cdot \mu(b, \pi)$ . But since  $b$ -signal workers must be paid the same wage in both sectors,  $w_b(\pi) = p(\pi) \cdot \mu(b, \pi) = 1$  we can pin down the price of good 1 as the inverse of the probability that a bad signal worker is productive in the high-tech sector,

$$p(\pi) = \frac{1}{\mu(b, \pi)} = \frac{\pi(1 - \eta) + (1 - \pi)\eta}{\pi(1 - \eta)} \quad (20)$$

The price must also satisfy a relationship imposed by demand shares (15):

$$p(\pi) = \frac{\alpha}{1 - \alpha} \frac{\overbrace{\underbrace{(1 - \beta)((1 - \eta)\pi + \eta(1 - \pi))}_{x_2 \text{ produced by } b\text{-workers}}}}{\underbrace{\eta\pi}_{x_1 \text{ produced by } g \text{ workers}} + \underbrace{\beta(1 - \eta)\pi}_{x_1 \text{ produced by } b \text{ workers}}} \quad (21)$$

Equating the right-hand sides of (20) and (21) determines the fraction of  $b$ -signal workers employed in Sector 1,  $\beta$ . The solution reveals that a positive  $\beta$  exists if and only if  $\alpha > \eta$ , as illustrated in Figure 3. We refer the reader again to the web appendix for details.

## 4.2 Equilibrium investments in Autarky

To obtain a closed form expression for the incentives to invest as a function of  $\pi$  substitute the wages and prices derived in Section 4.1 into (17). If  $\alpha \leq \eta$ , this function may be written as:<sup>17</sup>

$$B(\pi) = \max \left\{ (2\eta - 1) \left( \frac{\pi\eta}{\pi(1 - \eta) + (1 - \pi)\eta} \right)^\alpha \left( \frac{\alpha - (\pi\eta + (1 - \pi)(1 - \eta))}{\pi\eta + (1 - \pi)(1 - \eta)} \right), 0 \right\}. \quad (22)$$

Figure 4 plots  $B(\pi)$  for two sets of parameter values. All values where  $B(\pi) > 0$  in the figure correspond to type-A continuation equilibria, where  $g$  workers produce good 1 and  $b$  workers produce good 2.  $B(\pi)$  is single-peaked, but not necessarily concave (example in the right panel). Under different specifications of information and output technology the single-peakedness may break down, but what remains true is that the function is equal to zero at the extremes, and therefore must be initially increasing, and eventually decreasing. The reason is that if  $\pi = 0$  or  $\pi = 1$  workers are all equally productive in the production of both goods regardless of their signal (in particular, they are all unproductive in Sector 1 when  $\pi = 0$ ), therefore their wage does not depend on the signal. But if better signals are not rewarded with higher wages, incentives to invest are zero. Only when  $0 < \pi < 1$

<sup>17</sup>See the web appendix for a detailed derivation.

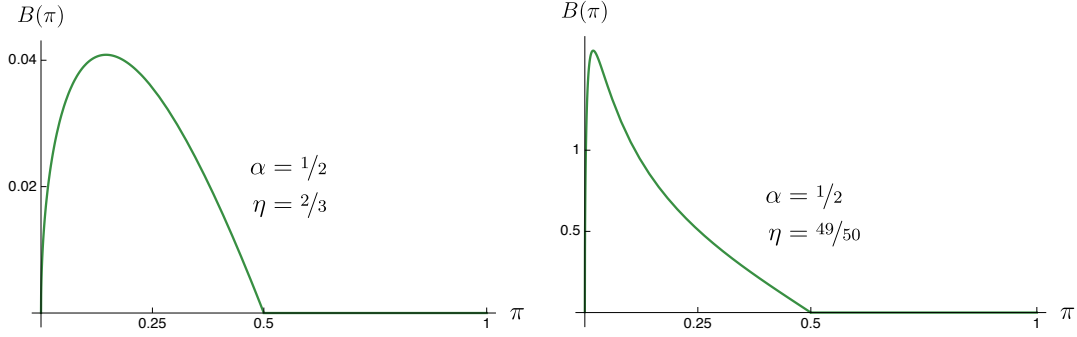


Figure 4: Gross incentives to invest under autarky

the signal carries information; workers that receive a good signal are paid higher wages, generating positive incentives to invest.

Any  $\pi$  such that  $\pi = G(B(\pi))$  is an equilibrium fraction of investors. Since  $G(B(\pi))$  is continuous and takes values on  $[0, 1]$ , existence follows trivially. The fixed point condition is illustrated in Figure 5, computed with  $\eta = 2/3$ ,  $\alpha = 1/2$  and  $G$  uniform over  $[\underline{c}, \bar{c}]$ , with  $\bar{c} - \underline{c} = 0.2$ . Changes in  $\underline{c}$  correspond to shifts in the cost distribution. If  $\underline{c} < 0$  (i.e. when some workers prefer to invest even without incentives) the equilibrium is unique. For  $\underline{c} = 0$ , there is a trivial equilibrium with no investment and an equilibrium with  $\pi > 0$ . As  $\underline{c}$  gets slightly larger there are three equilibria (one with  $\pi = 0$ ), whereas if  $\underline{c}$  is sufficiently large (not shown in the figure), as the curve shifts to the right only the trivial equilibrium with no investment remains.

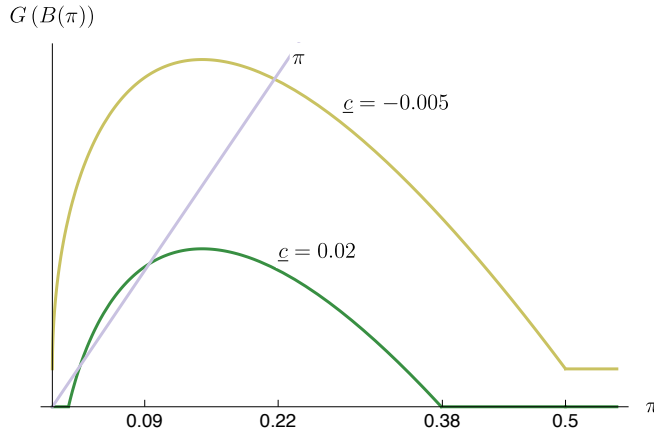


Figure 5: Equilibrium fixed point maps for two values of  $\underline{c}$ , with  $\eta = 2/3$ ,  $\alpha = 1/2$

In the next section we derive equilibria with two countries that trade. In many examples we will assume that a unique equilibrium with  $\pi > 0$  exists under autarky.<sup>18</sup> This is to

<sup>18</sup>Sufficient conditions are that  $G \circ B$  is concave and  $\underline{c} < 0$ . The first is a technical assumption needed



highlight that country specialization does not rely on multiplicity of equilibria, that is, on countries coordinating on different equilibria of the autarkic model (with multiplicity under autarky, further possibilities for specialization with trade arise). This assumption also eliminates “nuisance equilibria” with zero investments and makes welfare analysis sharper, not having to rely on comparisons between sets of equilibria.

## 5 Equilibria in the Trade Regime

In this section we assume that  $h$  and  $f$  trade on a frictionless world market. We will first prove by construction the main result of the paper: the existence of asymmetric equilibria with trade and specialization. Next, we provide some evidence of the generality of the result and show that trade equilibria exist even when there is a unique equilibrium without trade. While the replication of the autarky equilibrium in both countries remains an equilibrium of the two-country model (with no trade), we will show in the next section that this equilibrium may be unstable. We will conclude the analysis illustrating some welfare properties of the equilibria with trade.

### 5.1 Illustration of the existence of asymmetric equilibria by construction

The simplest asymmetric equilibrium we can construct occurs when the poor country, which we label as country  $h$ , is fully specialized in the low-tech sector. In such an equilibrium, the wage gap in  $h$  is zero, so the fraction of qualified workers in  $h$  is pinned down as  $\pi^h = G(0)$ . Then, the proportion of qualified workers in  $f$  solves a single variable fixed point equation similarly to the autarky case, but with some extra production of  $x_2$  performed in country  $h$ . Once  $\pi^f$  is obtained from this condition, it only remains to check that firms in  $h$  have no incentives to hire workers with signal  $g$  to produce the high-tech good.

To formalize the argument, assume  $G = U[0, 0.2]$ . Assuming all workers in country  $h$  specialize in the production of  $x_2$ , this induces zero incentives to invest, implying  $\pi^h = G(0) = 0$  and no incentives to place any worker in Sector 1 in country  $h$ . There is always a trivial equilibrium with  $\pi^f = 0$ , zero production of good 1, and zero utility for all, but we look for non-trivial equilibria with positive incentives to invest in  $f$ . If these equilibria exist, the equilibrium in country  $f$  is of type A or C (a fraction  $0 < \beta \leq 1$  of bad signal workers producing good 1).<sup>19</sup> The relative price of good 1 is pinned down by conditions similar to

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to ensure that  $G(\cdot)$  does not intersect the 45 degree line from below. The second assumption posits that an arbitrarily small fraction of workers like to make the investment even if there are no monetary gains. We do not believe this to be unrealistic.

<sup>19</sup>In equilibria of type B (mixing of good signals) some good signal workers produce good 2 and therefore receive wage 1, which is the same as the wage of bad signal workers. This provides no incentives to invest

the autarky case, but modified to take into account the production of good 2 occurring in country  $h$ . The equivalent of (21) is:<sup>20</sup>

$$p(\pi^f) = \frac{\overbrace{\lambda^f \left( (1-\beta)(1-\eta)\pi^f + \eta(1-\pi^f) \right)}^{x_2 \text{ produced in } f} + \overbrace{\lambda^h}^{x_2 \text{ in } h}}{\underbrace{\lambda^f \left( \eta\pi^f + \beta(1-\eta)\pi^f \right)}_{x_1 \text{ produced in } f}} \quad (23)$$

Where  $\beta = 0$  if the equilibrium is of type A (no workers with signal  $b$  produce good 1) and  $0 < \beta < 1$  if the equilibrium is of type C (some  $b$ -signal workers produce good 1). In a type-A equilibrium this equation defines the relative price of good 1, whereas if the equilibrium is of type C, this equation defines  $\beta$ : because  $b$  workers are employed in both sectors, the price is determined by equalizing their marginal productivity in the two sectors:  $p(\pi^f)\mu(b, \pi^f) = 1$ .

To derive incentives to invest, we now make two additional assumptions that do not hinder the generality of the result, as we discuss below, but simplify the derivations: we set equal Cobb-Douglas shares  $\alpha = 1/2$ , information technology parameter  $\eta = 2/3$ , and equal country sizes:  $\lambda^h = \lambda^f = 1/2$ . It is possible to show after simple but tedious algebraic simplifications, which we relegate to the web appendix, that the continuation equilibrium in country  $f$  is of type C. Workers with signal  $b$  in  $f$  are employed in both sectors, therefore the price is pinned down by equating the marginal product of these workers in the two sectors  $1 = p(\pi^f)\mu(b, \pi^f)$ , which, using (4), and  $\eta = 2/3$  implies  $p(\pi^f) = (2 - \pi^f) / \pi^f$ . Wages are:

$$w_b^f = 1, \quad w_g^f = p(\pi^f)\eta(g, \pi^f) = \frac{2 - \pi^f}{\pi^f} \frac{2\pi^f}{1 + \pi^f}.$$

Solving (23) for  $\beta$ , the fraction of  $b$ -signal workers in country  $f$  employed in Sector 1 is  $\beta = (1 + \pi^f) / (4 - 2\pi^f)$ . We are now in a position to derive incentives to invest in country  $f$ . We substitute our derivations into (17) to obtain,

$$B^f(\pi^f) = \frac{1}{6} \left( \sqrt{p(\pi^f)\mu(g, \pi^f)} - \frac{1}{\sqrt{p(\pi^f)}} \right) = \left( \frac{4 - 2\pi^f}{1 + \pi^f} - 1 \right) \sqrt{\frac{\pi^f}{2 - \pi^f}}, \quad (24)$$

with  $\mu(g, \pi) = 2\pi / (1 + \pi)$  from (4). Note that (24) is equal to zero for  $\pi^f = 0$  or 1. The equilibrium in country  $f$  is defined by the fixed-point equation  $\pi^f = G(B^f(\pi^f))$  with one interior solution at  $\pi^f = 0.49$  with  $p = 3.095$ . As will be shown next, this type of trade equilibrium is robust to perturbations of the parametric assumptions we made.

leading to the uninteresting equilibrium  $(\pi^h, \pi^f) = (0, 0)$ .

<sup>20</sup>Since we are looking for equilibria where  $\pi^h = 0$  we can drop the dependency of the price on  $\pi^h$ .

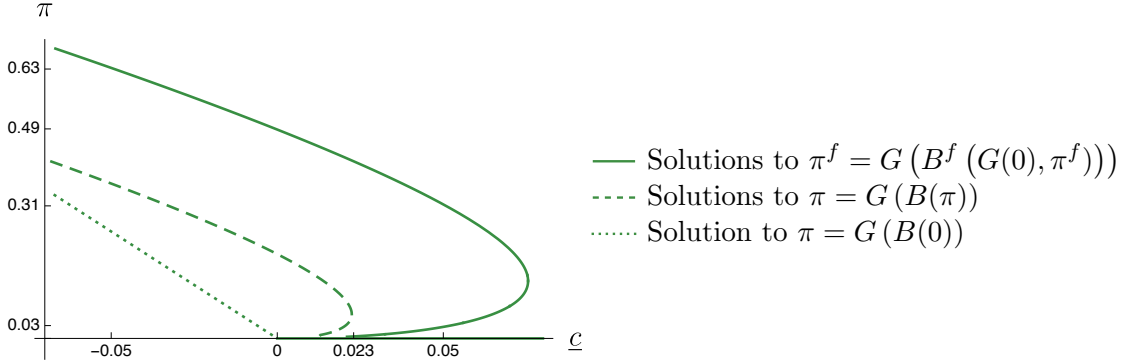


Figure 6: Equilibrium investments under trade with  $\eta = 2/3, \alpha = 1/2$  for different values of  $\underline{c}$ .

## 5.2 Robustness of the equilibria with trade

**The cost distribution.** We explore first how shifts in the cost distribution affects the existence of asymmetric equilibria of the type we computed in the previous subsection (full specialization of  $h$  country workers). We assume a uniform  $G$  over  $[\underline{c}, \underline{c} + 0.2]$ , and treat the lower bound of the distribution  $\underline{c}$  as a variable, holding the other parameters fixed.

Figure 6 illustrates the results. The solid line represents equilibrium investments in country  $f$  if there were no incentives to invest in country  $h$ . The dotted line is the fraction that is willing to invest without incentives, and the line in between represents equilibrium investments in autarky. It cannot be seen in the figure, but it can be shown that  $\pi^h = G(B(0))$  is a best response given that the country  $f$  invests in accordance with the solid line, so country  $h$  investing in accordance with the solid and  $f$  in accordance with the dotted line is an equilibrium under trade.

Both curves bend backwards, so there is a range with multiple equilibria in the autarky model (see dashed line where  $\underline{c} > 0$ ). If  $\underline{c} > 0$  zero incentives in county  $h$  implies  $\pi^h = 0$ . As can be seen from the solid line bending backwards in this region, there are three best responses in country  $f$  to  $\pi^h = 0$ : one is the trivial equilibrium  $\pi^f = 0$  whereas two have positive investment. There is also a range to the right of approximately  $\underline{c} = 0.023$  where there are two non-trivial asymmetric trade equilibria, despite the unique autarky equilibrium being a trivial zero investment equilibrium (the dashed line can't be seen but it corresponds to the horizontal axis in this range). For example if  $\underline{c} = 0.05$ ,  $\pi^f = \{0, 0.03, 0.31\}$  are all best responses to  $\pi^h = 0$ .

Multiple autarky equilibria are not necessary for trade to occur. For  $\underline{c}$  approximately between -0.07 and 0 there is an asymmetric equilibrium with trade, and a unique autarky equilibrium. To illustrate one such equilibria, when  $\underline{c} = -0.05$ , 25 percent of workers from

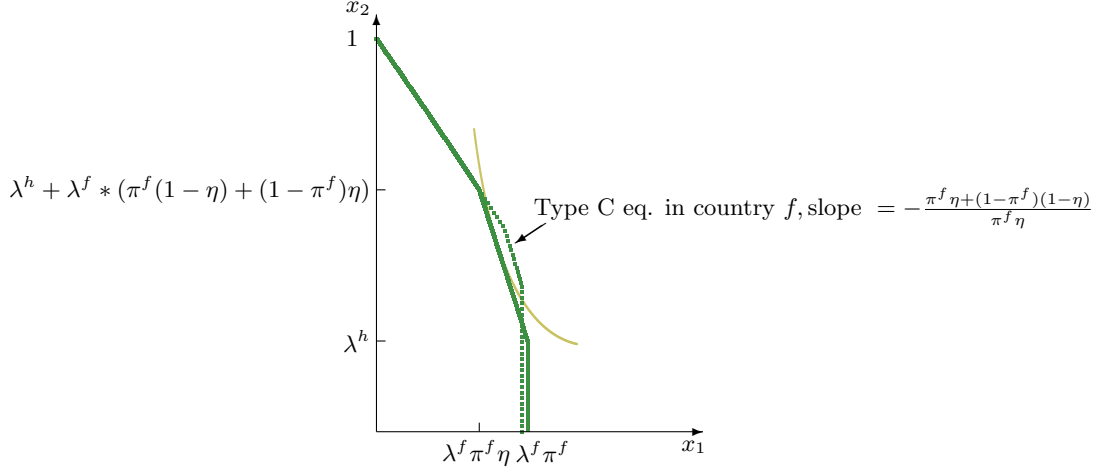


Figure 7: World production possibilities frontier when all in  $h$  produce good 2

country  $h$  are willing to invest even when there are no incentives to do so. Assuming that this is the case, and placing all workers of country  $h$  in Sector 2, in country  $f$  most workers specialize in Sector 1, generating incentives so that  $\pi^f = 0.63$  is the optimal response, with a relative price of good 1 equal to 2.16. It remains to be checked is that there are no incentives to employ country  $h$  workers with good signals in Sector 1. With  $\pi^h = .25$ , the expected probability of being qualified for a good worker is  $\mu(g, 0.25) = 0.4$ , which multiplied by the price 2.16 gives an expected productivity of 0.865, less than the unit productivity in Sector 2. In general, one can verify that this condition,  $p(\pi^f)\eta(g, \pi^h) \leq 1$ , is satisfied if  $\frac{4\pi^h}{1+3\pi^h} \leq \pi^f$ , which holds as long as  $\pi^h = G(0)$  is small enough. Indeed for lower values of the lower bound of the cost distribution not displayed in the figure, as the number of qualified workers in country  $h$  increases, it becomes impossible to sustain this type of asymmetric equilibrium.

**Country size and preference parameter.** Existence of this type of trade equilibria also does not hinge on our choice of the values of relative country size  $\lambda^h$  and of the Cobb-Douglas preference parameter  $\alpha$ . Figure 7 shows the production possibilities frontier when all workers in country  $h$  produce good 2. An increase in  $\lambda^h$  shifts the production possibilities frontier from the solid to the dotted line, but does not change the slope of the frontier in correspondence to a type-C equilibrium, because the relative productivity of workers in country  $f$  in the two sectors, determined by the information technology, does not change. Similarly, a change in  $\alpha$  changes the slope of the indifference curves. Therefore, perturbations of  $\alpha$  and  $\lambda^h$  (small enough so that the equilibrium remains of type C in country  $f$ ) change the point of tangency but not the equilibrium price, which is

defined by the slope of the production possibilities set.<sup>21</sup> Expected productivities, which are determined by the price and the information technology, do not change, therefore wages do not change. Incentives and equilibrium investment remain the same in both countries.

**Extreme specialization in country  $h$ .** Asymmetric equilibria also do not depend on the extreme specialization in country  $h$  we assumed to construct the equilibrium. The analysis gets more complicated because when positive incentives to invest exist in both countries, solving for equilibrium implies computing the solution to a system of two fixed-point equations. For an intuition, recall from Proposition 2 that the equilibrium price is decreasing in  $\pi^f$  (strictly, in some regions). From (17), incentives are increasing in price because price increases wages of  $g$ -signal workers more than wages of  $b$ -signal workers.<sup>22</sup> Hence, an increase in investments abroad decreases prices and incentives at home. Symmetrically, an increase in investments at home reduces incentives abroad. In reduced form, this is like a negative cross-country externality in human capital acquisition. These effects create equilibria where countries specialize: rich countries export the high-tech good and poor countries export the low-tech good, even when the autarky equilibrium is unique.

Formally, consider the region of the parameter space where equilibria are of type C or A in both countries,<sup>23</sup> so that  $w_g^j = p(\pi)\mu(g, \pi^j)$  and  $w_b^j = 1$ . Differentiate (17) with respect to the two countries' investment to obtain, using  $p$  as shorthand for  $p(\pi^h, \pi^f)$  and introducing notation  $\Psi = (2\eta - 1)\alpha^\alpha(1 - \alpha)^{1-\alpha}$ :

$$\frac{\partial B^f(\pi^h, \pi^f)}{\partial \pi^f} = \underbrace{\Psi p^{1-\alpha} \frac{d\mu(g, \pi^f)}{d\pi^f}}_{\text{"information effect"}} + \underbrace{\Psi p^{-\alpha} \left( (1 - \alpha)\mu(g, \pi^f) + \frac{\alpha}{p} \right) \frac{\partial p}{\partial \pi^f}}_{\text{"price effect"}} \quad (25)$$

$$\frac{\partial B^h(\pi^h, \pi^f)}{\partial \pi^f} = \underbrace{\Psi p^{-\alpha} \left( (1 - \alpha)\mu(g, \pi^f) + \frac{\alpha}{p} \right) \frac{\partial p}{\partial \pi^f}}_{\text{"price effect"}} \quad (26)$$

The price effect labeled in the equations is, as discussed, negative, and occurs in both countries whereas the information effect bites only in the country where investment changes. The information effect is positive because as the proportion of investors increases, the prob-

<sup>21</sup>Prices are constant because of the simplifying assumption that information technology has only two signals available. With a more general information structure the production possibilities set would be strictly convex, and small perturbations of  $\lambda^h$  or  $\alpha$  would have a small effect on equilibrium prices. To make the case that a nearby trade equilibrium still exists we would have to rely on continuity arguments.

<sup>22</sup>Either  $b$  workers are employed only in Sector 2, in which case their wage is fixed at 1, or some are employed in Sector 1, in which case their wage is  $p(\pi)\mu(b, \pi^j)$  which is less than the wage of  $g$ -signal workers employed in Sector 1,  $p(\pi)\mu(g, \pi^j)$ .

<sup>23</sup>This is necessary to have strictly positive incentives to invest in both countries

ability that an individual with good signal is productive increases as well, but its size depends on the size of  $\pi^f$ . Hence, starting from a non-trivial autarky equilibrium in which  $\pi^A = \pi^f = \pi^h$ , an increase in  $\pi^f$  either decreases function  $B^h$  and increases  $B^f$ , or it shifts  $B^h$  downwards more than it shifts  $B^f$ . A decrease in  $\pi^h$  has the symmetrically opposite effect. These derivations illustrate why the informational externality pushes countries to specialize. One can then find values  $\pi^h < \pi^f$  such that  $B^h(\pi^h, \pi^f) < B^f(\pi^h, \pi^f)$ . Whether these values satisfy the equilibrium conditions depends on the cost distribution, but examples can be constructed to this end.<sup>24</sup>

## 6 Stability and Welfare

### 6.1 Stability

A symmetric equilibrium replicating autarky always exists in the trade regime. However, this equilibrium can be unstable when the economy is open for trade.<sup>25</sup>

Consider a parameterization where  $\pi^A$  is a stable autarky equilibrium.<sup>26</sup> It follows that  $\pi = (\pi^A, \pi^A)$  is an equilibrium when the countries are allowed to trade.

We want to analyze the effects of small deviations from the symmetric equilibrium. Consider the change in relative price first. When  $\pi^h = \pi^f = \pi$  and assuming again  $\eta = 2/3$  and  $\alpha = 1/2$ , we are in the region where  $\eta \geq \alpha$ . The autarky equilibrium must be of type A. One can derive that when the equilibrium is of type A in both countries, the price is equal to  $p(\pi^h, \pi^f) = (4 - \pi^h - \pi^f)/2(\pi^h + \pi^f)$ ,<sup>27</sup> therefore  $p(\pi, \pi) = (2 - \pi)/2\pi$ , which is consistent with (18). Differentiating these expression gives:

$$\begin{aligned} \frac{d}{d\pi} p(\pi, \pi) &= \frac{-1}{(\pi)^2} && \text{(relevant under autarky)} \\ \frac{\partial}{\partial \pi^f} p(\pi^h, \pi^f) &= \frac{-2}{(\pi^h + \pi^f)^2} && \text{(relevant with trade).} \end{aligned} \tag{27}$$

Evaluating each expression at  $(\pi^A, \pi^A)$  we have that

<sup>24</sup>If one is willing to let the parameters of  $G$  be free, note for the sake of constructing a trade equilibrium that there is an infinite number of probability distributions satisfying the three restrictions on their domain that are needed for  $(\pi^h, \pi^f)$  to hold as a trade equilibrium together with  $\pi^A$  as an autarky equilibrium:  $G(B^h(\pi^h, \pi^f)) = \pi^h$ ,  $G(B^f(\pi^h, \pi^f)) = \pi^f$ , and  $G(B(\pi^A, \pi^A)) = \pi^A$ .

<sup>25</sup>Because the model lacks real time, “stability” is a somewhat ad hoc criterion that corresponds to the adjustment dynamic where  $\pi_{t+1}^j = G(B^j(\pi_t^j, \pi_t^k))$ ,  $j, k = h, f$ ,  $j \neq k$  (or the natural continuous analogue). Embedding the model in an OLG framework one obtains such dynamic system if one assumes that employers can not differentiate between workers of different cohorts.

<sup>26</sup>For example, when  $\underline{c} < 0$ , we know there is a unique autarky equilibrium, which must be stable since  $G(B(\pi))$  must intersect the 45° line from above.

<sup>27</sup>See the web appendix for the detailed derivation.

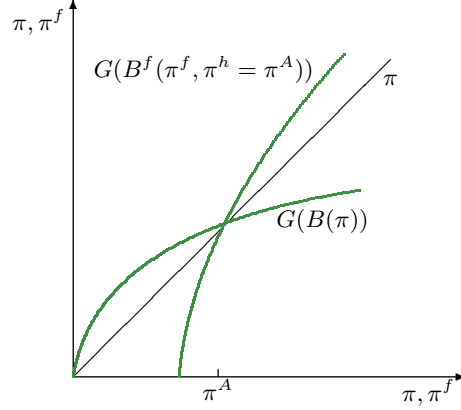


Figure 8: Best responses under trade and autarky, at the autarky equilibrium

$$\frac{d}{d\pi} p(\pi, \pi) \Big|_{\pi=\pi^A} - \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f} \Big|_{\pi^h=\pi^f=\pi^A} = \frac{-1}{(\pi^A)^2} - \frac{-2}{4(\pi^A)^2} = \frac{-1}{2(\pi^A)^2} < 0. \quad (28)$$

An increase in investments thus has a larger negative impact on the price in autarky, as intuition suggests. Autarky is equivalent to the trade regime with the added restriction that  $\pi^h = \pi^f = \pi$ . We compare the effect of a change in investment on incentives to invest (17) between the regimes. In the autarky case, we restrict the two arguments of  $B^f$  to be equal, while the second argument is unrestricted in the open economy case. With  $\alpha = 1/2$  and  $\eta = 1/2$ , the derivative of the incentives function (25) further simplifies to obtain (using  $p^A$  as shorthand notation for  $p(\pi^A, \pi^A)$ ):

$$\begin{aligned} \frac{dB^j(\pi, \pi)}{d\pi} \Big|_{\pi=\pi^A} &= \underbrace{\frac{\sqrt{p^A}}{6} \frac{d\mu(g, \pi)}{d\pi} \Big|_{\pi=\pi^A}}_{\text{"information effect"}} + \underbrace{\frac{1}{12\sqrt{p^A}} \left( \mu(g, \pi^A) + \frac{1}{p^A} \right) \frac{dp(\pi, \pi)}{d\pi} \Big|_{\pi=\pi^A}}_{\text{"price effect"}} \quad (29) \\ \frac{\partial B^f(\pi^h, \pi^f)}{\partial \pi^f} \Big|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}} &= \underbrace{\frac{\sqrt{p^A}}{6} \frac{d\mu(g, \pi^f)}{d\pi^f} \Big|_{\pi^f=\pi^A}}_{\text{"information effect"}} + \underbrace{\frac{1}{12\sqrt{p^A}} \left( \mu(g, \pi^A) + \frac{1}{p^A} \right) \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f} \Big|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}}}_{\text{"price effect"}}, \quad (30) \end{aligned}$$

In each case, the effect on incentives is decomposed as a positive “information effect” and a negative “price effect”. The information effect in (29) is the same as in (30), but, by (28), the price effect is stronger in autarky, so the slope of  $B^f(\pi^f, \pi^h = \pi^A)$  exceeds the slope of the autarky benefits of investment  $B(\pi)$ , when evaluating both functions at  $\pi^A$  (see Figure 8). Hence, it is possible that  $G(B^f(\pi^f, \pi^h = \pi^A))$  intersects the 45° line from below at  $\pi^f = \pi^A$  even if  $G(B(\pi))$  intersects from above. Since the curve  $G(B^f(\pi^f, \pi^h = \pi^A))$  intersecting the 45° line from below is a *sufficient* condition for local instability this shows

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[-0.02, 0.18]$	Trade, Country $h$	Trade, Country $f$	Autarky
Equilibrium Investment	$\pi^h = 0.1$	$\pi^f = 0.548$	$\pi = .269$
Per Capita Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = 0.463$ $y_2^f = 0.226$	$y_1 = 0.179$ $y_2 = 0.577$
Per Capita Consumption	$x_1^h = 0.189$ $x_2^h = 0.5$	$x_1^f = 0.274$ $x_2^f = 0.726$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.090$	$B(\pi, \pi) = 0.034$
Gross expected utility	0.307	0.446	0.321
Expected utility net of inv. cost	0.308	0.427	0.319
Expected utility if invest	$0.307 - c$	$0.487 - c$	$0.346 - c$
Expected utility if don't invest	0.307	0.397	0.313
Wages	$w_g^h = 1$ $w_b^h = 1$	$w_g^f = 1.875$ $w_b^f = 1$	$w_g = 1.364$ $w_b = 1$
Expected Wage	1	1.452	1.154
Prices	$p_1 = 2.648$		$p_1 = 3.216$

Table 1: Trade and autarky equilibria in Example 1

that the autarky equilibrium may be destabilized by opening up for trade.<sup>28</sup>

Next, we illustrate some welfare properties of the equilibria with trade.

## 6.2 Example 1: specialization may be beneficial only to the rich country

Table 1 displays a parameterization where all country  $f$  citizens are better off in the asymmetric trade equilibrium than in the unique autarky equilibrium, and where all country  $h$  citizens are worse off in the asymmetric trade equilibrium than under autarky.<sup>29</sup>

Notice that the total world output of both goods is higher in the asymmetric equilibrium (see the second row of the table). While prohibitive trade barriers would make country  $h$  better off, it is also true that there exists transfer payments from  $f$  to  $h$  that can make both countries better off relative to the autarky equilibrium. Hence there are some productive gains from specialization despite the countries being fundamentally identical.

## 6.3 Example 2: specialization may make both countries better off

In this example trade makes both countries better off. For maximal simplicity we rig this example so that the “free rider problem” in human capital investments is so severe that

<sup>28</sup>Examples are easy to find. When  $c$  is uniformly distributed on  $[0, 2]$ , the unique (non-trivial) autarky equilibrium is  $\pi = .0067$ . The equilibrium where  $\pi^f = \pi^h = 0.067$  is unstable under trade, while an asymmetric equilibrium with  $\pi^f = .0283$ ,  $\pi^h = 0$  is stable.

<sup>29</sup>Although some agents change their investment behavior in the comparison across equilibria, this does not complicate Pareto comparisons. The crucial fact is that (in the example) both qualified and unqualified workers gain (lose) in country  $f$  ( $h$ ). All workers in the rich country have the option to invest as in the autarky equilibrium, so revealed preferences imply that all workers gain. Similarly, in the poor country all workers have the option to invest as in the trade equilibrium when in autarky, so again, by revealed preferences, all workers are better off in autarky.



$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[.04, .24]$	Trade, Country $h$	Trade, Country $f$	Autarky
Equilibrium Investment	$\pi^h = 0$	$\pi^f = 0.353$	$\pi = 0$
Production	$y_1^h = 0$	$y_1^f = 0.284$	$y_1 = 0$
	$y_2^h = 1$	$y_2^f = 0.323$	$y_2 = 1$
Consumption	$x_1^h = 0.107$	$x_1^f = 0.177$	$x_1 = y_1$
	$x_2^h = 0.5$	$x_2^f = 0.823$	$x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.111$	$B(\pi, \pi) = 0$
Gross average utility	0.232	0.381	0
Avg. utility net of inv. cost	0.232	0.355	0
Expected utility if invest	$0.232 - c$	$0.452 - c$	$0 - c$
Expected utility if don't invest	0.232	0.342	0
Wages	$w_g^h = 1$	$w_g^f = 2.433$	$w_g = -$
	$w_b^h = 1$	$w_b^f = 1$	$w_b = 1$
Expected Wage	1	1.647	1
Prices	$p_1 = 4.660$		$p_1 = -$

Table 2: Trade and autarky equilibria in Example 2

the unique equilibrium under autarky is the trivial equilibrium. However, with trade, the existence of the other country means that, for any investment  $\pi^f$  in country  $f$ , the price of good 1 is higher than without trade if there is no human capital investment in country  $h$ . Hence, trade allows a new market to emerge that would not operate without trade.

In Table 2 we summarize one example where the market for good 1 only operates with international trade. There are multiple trade equilibria and the numbers in the table refer to the equilibrium with the largest fraction of investors in the country producing good 1.<sup>30</sup>

Consumers are happier when consuming both goods than when consuming only one good. Because a new market opens up, trade is beneficial for both countries.

## 6.4 Pareto Improving Inequality

The example presented above is extreme, but specialization through trade may more generally be viewed as an imperfect “solution” to the informational problem in the model.<sup>31</sup> In the example, there is no way for a market to open unless the rewards for getting into the market are large enough. These rewards are bigger if only one country enters the market: the same “kick” from the local informational externality is generated at a smaller negative price effect. Specialization thus reduces the problem of under investment in human capital.

Even in less extreme cases, both countries may gain from specializing. As is illustrated in Figure 9 it is *always* true that the production possibilities set expands when moving from a situation where both countries invest at the same rate to an asymmetric investment profile for a constant total quantity of investors in the world. In the figure, the frontier to

<sup>30</sup>There is also an equilibrium with  $\pi^h = 0, \pi^f = 0.0157$ . Unlike the equilibrium in Table 2 this is unstable.

<sup>31</sup>For a detailed elaboration on this point in the context of discrimination, see Norman (2003).

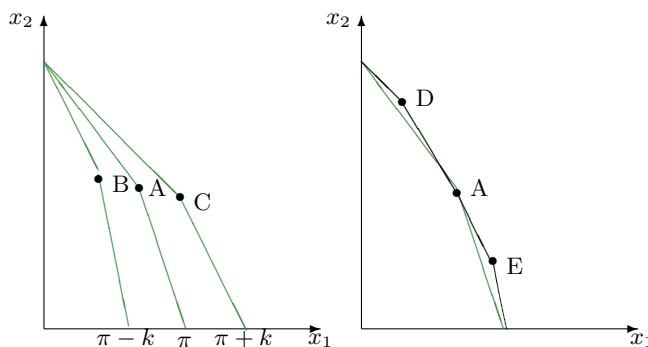


Figure 9: Specialization Expands the World Production Possibilities

the left with the kink at point  $A$  is some symmetric investment profile, whereas the frontiers with kinks at  $B$  and  $C$  corresponds with an asymmetric investment profile. Assuming that countries are of equal size, the total number of investors in the world is unchanged, but the world production possibilities set is nevertheless larger (the frontier with kinks at  $D$ ,  $A$  and  $E$  in the graph to the right). To understand this, note that the efficient way of increasing  $x_1$  starting from the vertical intercept is to first only use workers from the country with investments  $\pi+k$  with good signals, so initially the slope of the world production possibilities set must be the same as the set to the left with kink at  $C$ . The graph is drawn for the case where it is better to use high-signal workers from the low investment country than low signal workers from the high investment country in Sector 1, but the result is fully general.

## 6.5 The Irrelevance of Size

Since this is a general equilibrium model with large countries, changes of the relative size of the countries will in general affect the *asymmetric* equilibria due to price effects. The nature of such changes depends on the parameterization. For example, if the example in Section 6.3 is extended to allow for different country sizes, there is a critical size such that the country must fully specialize in the low-tech industry if it exceeds this critical size. Reducing the size of the country from  $1/2$  on the other hand only improves incentives. Hence, there are circumstances where *the only asymmetric equilibrium* is that the small country becomes rich. It is also possible to set up examples that go the other way, where only the big country can end up on top (see Appendix A.2).

However, these are not really “country-scale-effects”. Instead, we prefer to think of them as scale effects that depend on relative size of the North to the South. To understand, suppose that there are  $n$  countries indexed by  $j \in \{1, \dots, n\}$ , of size  $\lambda^j$ . Consider an equilibrium in this model where the set of countries is partitioned into the sets  $P$  and  $R$  and where

$\pi^j = \pi^p$  for all  $j \in P$  and  $\pi^j = \pi^r$  for all  $j \in R$ . Finally let  $\lambda^p = \sum_{j \in P} \lambda^j$  and  $\lambda^r = \sum_{j \in R} \lambda^j$ . This is an equilibrium if and only if  $(\pi^p, \pi^r)$  is an equilibrium in the two-country model with countries of sizes  $(\lambda^p, \lambda^r)$ . There may of course be other equilibria as well, but at least for this form of equilibrium the size of the *individual country* is irrelevant and the relevant scale effect can be interpreted in our preferred manner.

A “development miracle” can be interpreted as a country that manages to re-coordinate from being part of the developing world to being part of the developed world. The model cannot explain how such a re-coordination is achieved, but, if the economy is small, the effects on the rest of the world are negligible. In contrast, a simultaneous re-coordination of a significant fraction of the “South” may lead to large enough relative price changes so that it is not worth the while as long as there is no change in the “North”. Obviously, the model is too stylized for policy recommendations, but this nevertheless suggests that it may be misguided to use a few small successful countries as a model for all developing countries.

## 7 Concluding Remarks

This paper shows that it is possible to generate endogenous comparative advantages between identical countries in an essentially neoclassical model. Specialization and trade arise due to an informational externality: workers are better informed than firms about their abilities.

Two-country model equilibria can be reinterpreted as  $n$ -country model equilibria where countries cluster in two groups in terms of level of development. Equilibria of the  $n$ -country model are neutral with respect to the size of individual countries, so the model is consistent with a world with no particular relationship between size and the level of development.

A natural extension is to introduce physical capital into the production technology. This would be interesting for analyzing the role of foreign capital and capital flight from poor countries. As this paper focuses on the effects asymmetric information about skills we have chosen to ignore physical capital. However, if capital and human capital were complementary in production, the effects analyzed in this paper would be reinforced.

To understand, suppose initially that capital cannot flow between countries. Except for a capital market equilibrium condition the model is more or less the same as the one without capital. Consider an asymmetric equilibrium under the assumption that initial capital endowments are identical. As capital is more useful in the high-tech industry the return on capital is higher in the rich country, so, with free capital mobility, the rich country must have a higher per capita level of capital. Notice that the movement of capital from the poor to the rich country affects incentives to invest positively in the rich country and

negatively in the poor country, strengthening the incentives to specialize.<sup>32</sup>

Because this is a static model, we do not analyze workers' incentives to migrate. Workers with good signals in poor countries may find it advantageous to migrate where their skills receive better rewards. However, such incentives are mitigated if employers recognize the workers' country of origin. When a foreign country employer forms beliefs about a home country worker's productivity, she may take into account the worker's nationality, therefore the expected productivity computed by foreign and home country employers is the same. In this case, incentives to acquire human capital are defined by citizenship, not residence.<sup>33</sup>

## A Appendix

### A.1 Proof of proposition 1.

(Part 1) Consider an arbitrary equilibrium. Let  $x^* = (x_1^*, x_2^*)$  denote the world production, where  $x_i^* = \lambda^h x_i^{h*} + \lambda^f x_i^{f*}$  and  $x_i^{j*}$  denotes the production of good  $i$  in country  $j$  in equilibrium. Also let  $l_i^{j*}(\theta)$  denote the corresponding input of workers with signal  $g$  in economy  $j$  and sector  $i$ . By profit maximization  $p_i^* x_i^{j*} - \sum_{\theta \in g, b} w_\theta^{j*} l_i^{j*}(\theta) \geq p_i^* x_i^{j'} - \sum_{\theta \in g, b} w_\theta^{j*} l_i^{j'}(\theta)$  for any alternative plan  $(x_i^{j'}, l_i^{j'}(\cdot))$ . Adding over the two sectors and imposing the market clearing conditions on the labor market we conclude that  $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*} (\eta \pi^j + (1 - \eta)(1 - \pi^j)) - w_b^{j*} ((1 - \eta) \pi^j + \eta(1 - \pi^j)) \geq \sum_{i=1,2} p_i^* x_i^{j'} - w_g^{j*} (l_1^j(g) + l_2^j(g)) - w_b^{j*} (l_1^j(b) + l_2^j(b))$  for all possible alternative production plans (feasible as well as non-feasible in the aggregate). Now for any feasible alternative allocation  $l_1^j(g) + l_2^j(g) \leq \eta \pi^j + (1 - \eta)(1 - \pi^j)$  and  $l_1^j(b) + l_2^j(b) \leq (1 - \eta) \pi^j + \eta(1 - \pi^j)$ , implying that  $\sum_{i=1,2} p_i^* x_i^{j*} \geq \sum_{i=1,2} p_i^* x_i^{j'}$  for any feasible alternative  $(x_1^{j'}, x_2^{j'})$ . Since this must hold in each country we conclude that  $p^* x^* \geq p^* x'$  for any alternative feasible world production vector  $x' = (x_1', x_2')$ . Moreover, in order for  $(x_1^{j*}, x_2^{j*})$  to be profit maximizing it must be that  $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*} (\eta \pi^j + (1 - \eta)(1 - \pi^j)) + w_b^{j*} ((1 - \eta) \pi^j + \eta(1 - \pi^j)) = 0$ . Finally, since  $u$  is homothetic it follows from standard arguments that if  $(x_1^{j*}(w), x_2^{j*}(w))$  solves the utility maximization problem for a worker with income  $w$ , then  $(\frac{w'}{w} x_1^{j*}(w), \frac{w'}{w} x_2^{j*}(w))$  solves the utility maximization problem for a worker with income  $w'$ . Consider the program

$$\begin{aligned} & \max_{x_1, x_2} u(x_1, x_2) & (A1) \\ \text{s.t.} & p_1^* x_1 + p_2^* x_2 \leq p_1^* x_1^* + p_2^* x_2^* = p_1^* \sum_{j=h,f} \lambda^j x_1^{j*} + p_2^* \sum_{j=h,f} \lambda^j x_2^{j*}, \end{aligned}$$

<sup>32</sup>Details are available on request from the authors.

<sup>33</sup>However, employers may also condition their beliefs on migration status, which is as easily recognizable as citizenship. This raises the possibility that migrants acquire a reputation for higher investment than their fellow citizens who did not migrate, and that this belief is confirmed in equilibrium.

where the star-superscript refers to equilibrium variables. The aggregate consumption bundle of any equilibrium must be a solution to (A1) because the problem gets the relative consumptions of  $x_1$  and  $x_2$  right and that  $p_1^*x_1^* + p_2^*x_2^*$  is the aggregate world income. We thus conclude that if  $x^*$  is an equilibrium world consumption plan it must solve (A1). Since the set  $X^W(\pi^h, \pi^f)$  is contained in the “budget set” of representative and  $x^* \in X^W(\pi^h, \pi^f)$  it follows that  $x^*$  must be a solution to (14).

(Part 2) Let  $x^*$  solve (14) and let  $V = \{x \in R_+^2 | u(x) > u(x^*)\}$ . Quasi-concavity implies that  $V$  is a convex set. The set  $X^W(\pi^h, \pi^f)$  is also convex (see Page 11). Moreover,  $V \cap X^W(\pi^h, \pi^f) = \emptyset$ , so the separating hyperplane theorem (Theorem 11.3. in Rockafellar (1997)) implies that there exists some  $p^*$  such that  $p^*x \geq p^*x^*$  for all  $x \in V$  and  $p^*x \leq p^*x^*$  for every  $x \in X^W(\pi^h, \pi^f)$ . Let the wages be given by  $w_g^{j*} = \max\{p_1^*\mu(g, \pi^j), p_2^*\}$  and  $w_b^{j*} = \max\{p_1^*\mu(b, \pi^j), p_2^*\}$ , and let the allocation of workers be as in the planning solution. Observe in particular that if  $p_1^*\mu(\theta, \pi^j) > p_2^*$ , then no worker with signal  $\theta$  is employed in Sector 2 in the allocation that produces  $x^*$ . This is most easily seen in the differentiable case, where the optimality condition to (14) implies that  $\frac{\partial u(x^*)}{\partial x_1^*} / \frac{\partial u(x^*)}{\partial x_2^*} = \frac{p_1^*}{p_2^*} > \frac{1}{\mu(g, \pi^j)}$ . But,  $\frac{1}{\mu(\theta, \pi^j)}$  is the cost of producing an extra unit of good 1 by giving up some country  $j$  workers with good signal currently in production of good 2, so we conclude that as if representative consumer would be better off if some of these workers would be switched to the production of good 1, contradicting optimality of  $x^*$  if  $p_1^*\mu(\theta, \pi^j) > p_2^*$  and some of the  $j$  workers are assigned to Sector 2. A symmetric argument holds for when the inequality is reversed. Hence, if  $l_1^{*j}(\theta) > 0$ , then  $p_1^*\mu(\theta, \pi^j) = \max\{p_1^*\mu(\theta, \pi^j), p_2^*\} = w_\theta^{j*}$ , implying that the profit from hiring any quantity workers with signal  $\theta$  is zero in Sector 1, whereas if  $l_1^{*j}(\theta) = 0$ , then  $p_1^*\mu(\theta, \pi^j) \leq \max\{p_1^*\mu(\theta, \pi^j), p_2^*\} = w_\theta^{j*}$ , so no gain can be earned from hiring a positive quantity. The argument for Sector 2 is symmetric, which leads us to conclude that the outputs and (implicit) allocation of workers in the solution to (14) are consistent with profit maximizing behavior given the prices and wages constructed. ■

## A.2 Scale Effects in the Two Country Model

We construct examples showing that scale effects may go either way. One way is to look at the extreme case where  $\lambda^h$  is near zero. Equilibria can be calculated by solving two separate (different) one-dimensional fixed point problems. Consider the incentives to invest in a country with fraction of investors  $\pi$  under the “small open economy” assumption that the price (of good 1) is fixed at  $p$ . Equilibrium wages in the small open economy are determined to generate zero profits:  $w_g^O = \max\{p\mu(g, \pi), 1\}$  and  $w_b^O = \max\{p\mu(b, \pi), 1\}$ .

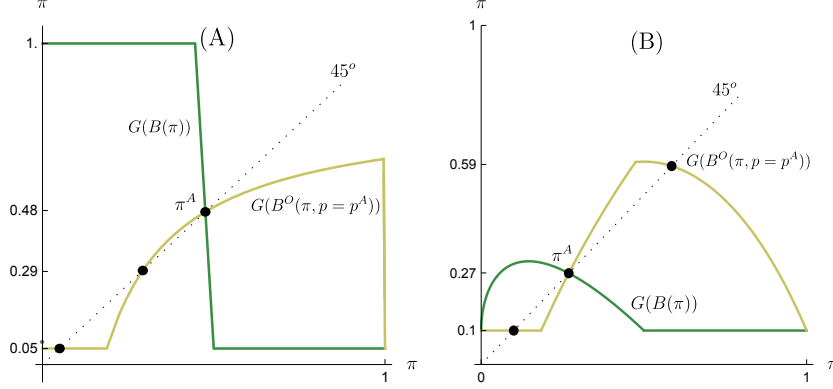


Figure 10: Equilibrium fixed point maps in a small open economy

The incentive to invest in the small open economy, denoted  $B^O(\pi; p)$ , is thus (using (17)),

$$B^O(\pi; p) = \frac{(2\eta - 1)\alpha^\alpha(1 - \alpha)^{1-\alpha}}{p^\alpha} \max\{\max\{p\mu(g, \pi), 1\} - \max\{p\mu(b, \pi), 1\}, 0\}. \quad (\text{A2})$$

If  $p^A$  is well defined (i.e., whenever the autarky equilibrium is non-trivial), then

$$\begin{aligned} B^h(\pi^h, \pi^A) &\rightarrow B^O(\pi^h, p = p^A) \text{ for all } \pi^h \in [0, 1] \text{ as } \lambda^h \rightarrow 0 \\ B^f(\pi^h, \pi^A) &\rightarrow B(\pi^f) \text{ for all } \pi^f \in [0, 1] \text{ as } \lambda^h \rightarrow 0, \end{aligned} \quad (\text{A3})$$

Assume parameters implying a unique autarky equilibrium, and call it  $\pi^A$ . Let  $p^A$  denote the associated autarky price. If  $\pi = \pi^A$ , then  $B^O(\pi^A; p = p^A) = B(\pi^A)$ , and  $\pi^A$  solves

$$\pi = G(B^O(\pi; p = p^A)). \quad (\text{A4})$$

While both (A4) and the autarky fixed point equation have  $\pi^A$  as a common solution, incentives diverge for other values of  $\pi$  since in autarky the price changes as  $\pi$  changes whereas there are no such price effects in (A2). Equation (A4) will therefore in many cases have solutions different from  $\pi^A$ . Now, if  $\pi^O$  solves (A4) and if  $\frac{d}{d\pi}\big|_{\pi=\pi^A} [\pi - G(B(\pi))] \neq 0$  and  $\frac{d}{d\pi}\big|_{\pi=\pi^O} [\pi - G(B^O(\pi; p = p^A))] \neq 0$ , then, for  $\lambda^h$  small enough, there exists an equilibrium  $(\pi^{h*}, \pi^{f*})$  in the trade model near  $(\pi^O, \pi^A)$ .<sup>34</sup>

We computed two examples to show that scale effects are indeterminate. Figure 10 panel (A) (computed using  $\alpha = 1/2$ ,  $\eta = .97$ ,  $\underline{c} = -0.005$ ,  $\bar{c} = .995$ ) illustrates the case where only the big country can be rich. There is a unique symmetric equilibrium at  $\pi^A = 0.48$ ; (A2)

<sup>34</sup>The slope condition for the autarky equilibrium is satisfied under the conditions that guarantee the equilibrium uniqueness. Its role is that if the equilibrium was at a tangency with the 45° line, the slightest effect from abroad could eliminate the equilibrium.

intersects the  $45^0$  line only below  $\pi^A$ . There are two asymmetric equilibria where the big country invests at  $\pi^h = \pi^A$  and the small country at  $\pi^f = 0.05$  or  $0.29$ . Figure 10 panel (B) was computed with the parameters as in Numerical Example 1. Both  $(\pi^h, \pi^f) = (.27, 0.1)$  and  $(\pi^h, \pi^f) = (.27, 0.59)$  are equilibria, so the small country can be either richer or poorer than the big country. Finally, when the unique autarky equilibrium is at  $\pi^A = 0$ , if the large country is large enough only the small country can be richer. Taken together, these three cases imply that there may be scale effects in favor of either the larger or the smaller economy, and that sometimes the equilibrium selection matters.

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