# The Effect of Statistical Discrimination on Black-White Wage Inequality: Estimating a Model with Multiple Equilibria - Omitted Computations

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This external appendix contains the detailed formula of the derivative of the wage function necessary to compute the likelihood in step 1 of the estimation strategy,and a description of the computation of the parameters of the production function and moments of skill endowment in step 2 of the estimation strategy.

All references to equation identifiers not beginning with a letter refer to the equations in the main paper.

## A Derivative of the wage function

In (26) we have $\frac{dt^j(\omega|E^j)}{d\omega} = \frac{1}{\frac{dw^j(\theta_i|E^j)}{d\theta_i}}$. This is the formula to compute the derivative.

If $\theta < \widetilde{\theta}^j$ :

$$
\begin{aligned}
\frac{dw^j(\theta_i|E^j)}{d\theta_i} = \; & (\pi^j \cdot y_k^j \cdot (\gamma^j - 1) \cdot \theta\hat{}(\gamma^j - 2) - (1 - \pi^j) \cdot y_h^j \cdot (\gamma^j - 1) \cdot (1 - \theta)\hat{}(\gamma^j - 2))/ \\
& (\pi^j \cdot \theta\hat{}(\gamma^j - 1) + (1 - \pi^j) \cdot (1 - \theta)\hat{}(\gamma^j - 1)) - \\
& (\pi^j \cdot y_k^j \cdot \theta\hat{}(\gamma^j - 1) + (1 - \pi^j) \cdot y_h^j \cdot (1 - \theta)\hat{}(\gamma^j - 1)) \\
& \cdot(\pi^j \cdot (\gamma^j - 1) \cdot \theta\hat{}(\gamma^j - 2) - (1 - \pi^j) \cdot (\gamma^j - 1) \cdot (1 - \theta)\hat{}(\gamma^j - 2))/ \\
& (\pi^j \cdot \theta\hat{}(\gamma^j - 1) + (1 - \pi^j) \cdot (1 - \theta)\hat{}(\gamma^j - 1))\hat{}2
\end{aligned}
$$

else if $\theta \geq \widetilde{\theta}^j$ :

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$$
\begin{aligned}
\frac{dw^j(\theta_i|E^j)}{d\theta_i} &= \pi^j \cdot \left( y_k^j + y_h^j \frac{(1-\pi^j)\, f_u\left(\widetilde{\theta}^j\right)}{\pi^j f_q\left(\widetilde{\theta}^j\right)} \right) \cdot (\gamma^j - 1) \cdot \theta\hat{}(\gamma^j - 2)/ \\
& (\pi^j \cdot \theta\hat{}(\gamma^j - 1) + (1-\pi^j) \cdot (1-\theta)\hat{}(\gamma^j - 1)) - \\
& \pi^j \cdot \left( y_k^j + y_h^j \frac{(1-\pi^j)\, f_u\left(\widetilde{\theta}^j\right)}{\pi^j f_q\left(\widetilde{\theta}^j\right)} \right) \cdot \theta\hat{}(\gamma^j - 1) \cdot \\
& (\pi^j \cdot (\gamma^j - 1) \cdot \theta\hat{}(\gamma^j - 2) - (1-\pi^j) \cdot (\gamma^j - 1) \cdot (1-\theta)\hat{}(\gamma^j - 2))/ \\
& (\pi^j \cdot \theta\hat{}(\gamma^j - 1) + (1-\pi^j) \cdot (1-\theta)\hat{}(\gamma^j - 1))\hat{}2
\end{aligned}
$$

# B How to compute the parameters of the production function, marginal products and skill moments.

Rewrite the equations in (27), together with the expression for $C^j$ and $S^j$

$$
\begin{aligned}
y_k^b + y_h^b \frac{1-\pi^b}{\pi^b} \left( \frac{1-\widetilde{\theta}^b}{\widetilde{\theta}^b} \right)^{(\gamma^b - 1)} &= \rho^b y_1(\cdot) \overline{e_q^b} && \text{(B1)} \\
y_k^w + y_h^w \frac{1-\pi^w}{\pi^w} \left( \frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w} \right)^{(\gamma^w - 1)} &= y_1(\cdot) \overline{e_q^w} && \text{(B2)} \\
y_h^b &= y_2(\cdot) \overline{e_u^b} && \text{(B3)} \\
y_h^w &= y_2(\cdot) \overline{e_u^w} && \text{(B4)} \\
y_k^b &= y_2(\cdot) \overline{e_q^b} && \text{(B5)} \\
y_k^w &= y_2(\cdot) \overline{e_q^w} && \text{(B6)} \\
y_1 &= \alpha C^{\alpha-1} S^{1-\alpha} && \text{(B7)} \\
y_2 &= (1-\alpha) C^{\alpha} S^{-\alpha} && \text{(B8)} \\
C^b &= \lambda^b \pi^b \rho^b (1 - F_q^b(\widetilde{\theta}^b)) \overline{e_q^b} && \text{(B9)} \\
C^w &= \lambda^w \pi^w (1 - F_q^w(\widetilde{\theta}^w)) \overline{e_q^w} && \text{(B10)} \\
S^b &= \lambda^b \left( \pi^b F_q^b(\widetilde{\theta}^b) \overline{e_q^b} + ((1-\pi^b) F_u^b(\widetilde{\theta}^b) \overline{e_u^b} \right) && \text{(B11)} \\
S^w &= \lambda^w \left( \pi^w F_q^w(\widetilde{\theta}^w) \overline{e_q^w} + ((1-\pi^w) F_u^w(\widetilde{\theta}^w) \overline{e_u^w} \right) && \text{(B12)}
\end{aligned}
$$

From B3/B4:

$$
\overline{e_u^w} = \frac{y_h^w}{y_h^b} \overline{e_u^b}
$$

From B3/B5:

$$
\begin{aligned}
\overline{e_u^b} &= \frac{y_h^b}{y_k^b} \overline{e_q^b} \\
&\rightarrow \overline{e_u^w} = \frac{y_h^w}{y_k^b} \overline{e_q^b}
\end{aligned}
$$

From B6/B5:

$$\overline{e_q^w} = \frac{y_k^w}{y_k^b}\overline{e_q^b}$$

Now averages $\overline{e_u^w}, \overline{e_u^b}, \overline{e_q^w}$ are a function of $\overline{e_q^b}$.

Get $\rho^b$ from B1/B2:

$$\rho^b = \frac{y_k^b + y_h^b \frac{1-\pi^b}{\pi^b}\left(\frac{1-\widetilde{\theta}^b}{\widetilde{\theta}^b}\right)^{(\gamma^b-1)}}{y_k^w + y_h^w \frac{1-\pi^w}{\pi^w}\left(\frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w}\right)^{(\gamma^w-1)}}\frac{\overline{e_q^w}}{\overline{e_q^b}} = \frac{y_k^b + y_h^b \frac{1-\pi^b}{\pi^b}\left(\frac{1-\widetilde{\theta}^b}{\widetilde{\theta}^b}\right)^{(\gamma^b-1)}}{y_k^w + y_h^w \frac{1-\pi^w}{\pi^w}\left(\frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w}\right)^{(\gamma^w-1)}}\frac{y_k^w}{y_k^b}$$

Substitute into B9-B12 and derive $S/C$ , $y_1$, and $y_2$ where $e_q^b$ cancels out.

From B2/B6 and B7/B8:

$$\begin{aligned}\frac{y_1}{y_2} &= \frac{y_k^w + y_h^w \frac{1-\pi^w}{\pi^w}\left(\frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w}\right)^{(\gamma^w-1)}}{y_k^w} = \frac{\alpha}{1-\alpha}\frac{S}{C} \\ \rightarrow \quad \alpha &= \frac{\frac{C}{S}\frac{y_k^w + y_h^w \frac{1-\pi^w}{\pi^w}\left(\frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w}\right)^{(\gamma^w-1)}}{y_k^w}}{1+\frac{C}{S}\frac{y_k^w + y_h^w \frac{1-\pi^w}{\pi^w}\left(\frac{1-\widetilde{\theta}^w}{\widetilde{\theta}^w}\right)^{(\gamma^w-1)}}{y_k^w}}\end{aligned}$$

Then get $y_1$ and $y_2$ since again $e_q^b$ factors out. From there get everything else.